Diversification Benefits of REIT Preferred and Common Stock: New Evidence from a Utility based Framework

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Abstract

We study the diversification benefits of REIT preferred and common stock using a utility based framework in which investors segment based on risk aversion. Taking the view of a long run investor, we conduct our analysis using data from 1992 to 2012. We examine optimal mean-variance portfolios of investors with different levels of risk aversion given access to different classes of assets and establish three main results. First, REIT preferred and common stock provides significant diversification benefits to investors. REIT common stock helps low risk aversion investors attain portfolios with higher returns, while REIT preferred stock helps high risk aversion investors by providing a venue for risk reduction. Both asset classes receive material allocations over plausible levels of risk aversion. Second, while REIT preferred stock appears to behave somewhat like a hybrid debt/equity asset, its risk/return profile appears to not easily be replicated by those asset classes. When given the opportunity, investors will reduce allocations to REIT common stock and investment grade bonds and invest in REIT preferred stock. Finally, realistic investor constraints matter empirically. Conclusions drawn from the empirical analysis are markedly different under these constraints compared to the classical unconstrained setting.

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1 Introduction

We employ a utility based approach to examine the diversification benefits of REITs. In the standard mean-variance framework where investors are unconstrained in their portfolio formation and can long and short the risk free asset, two fund separation obtains, and all investors hold portfolios that are a combination of the risk free asset and the market portfolio.[2] In this setting, maximizing the Sharpe Ratio of the market portfolio is equivalent to maximizing utility. Any asset that increases the Sharpe Ratio of the market portfolio is valuable to investors. However, investors do face short sales constraints and cannot borrow at the risk free rate. We show that, importantly, when investors are constrained in this way, the way we need to think about diversification benefits also changes. Investors no longer hold portfolios comprised of just the risk free asset and the market portfolio. Their portfolios differ based on their risk preferences. That is, investors segment based on risk aversion. In this setting, the question is not whether an asset provides diversification benefits; but, to whom does the asset provide diversification benefits?

Our motivation for employing a utility based approach to examine diversification is twofold. First, we know investors face constraints and cannot short all risky assets or borrow at the risk free rate.[3] Importantly, when these two constraints are combined, the usual two fund separation results from mean-variance analysis break down.[4] This means that in order to understand portfolio choice, the preferences of the investor need to be considered. Second, using the standard unconstrained mean-variance framework typically employed in the prior literature,[5] it is unclear that REITs provide any diversification benefits to investors when they are given anything but a very restricted set of assets to invest in.[6] It is possible that REITs as an asset class are important to investors, the literature has just been examining the issue in a framework that fails to detect these benefits.

To examine the benefits that REIT equities may provide to investors, we include both REIT common stock and REIT preferred stock in our analysis. To our knowledge we are the first study to examine the benefits of REIT preferred stock to investors. While preferred stock has received some attention in the academic literature, these papers tend to focus on either why firms choose to issue preferred stock over other classes of securities,[7] the role of preferred stock in the capital structure,[8] or on the pricing of preferred stock.[9] The issue we examine in this paper is why investors might choose to hold these securities. This is important because preferred stock forms a significant portion of REIT capital structure.[10] In equilibrium, for REITs to be able to raise capital

[2] See Markowitz (1952) for the original thesis without a risk free asset, Sharpe (1964) for the case that includes the risk free asset, and Merton (1972) for a mathematical characterization of the efficient frontier.
[3] See Geczy, Musto, and Reed (2002) and Bris, Goetzmann, and Zhu (2007) for a discussion of short sales constraints.
[4] The first constraint (the inability to short the risky assets) is considered by Ross (1977). The second constraint (the inability to borrow at the risk free rate) is considered by Black (1972). In both cases the solution to the portfolio problem takes the form of two fund separation. We discuss later how in the presence of both these constraints two fund separation breaks down.
[5] Sa-Aadu, Shilling, and Tiwari (2010) is an exception.
[6] See Chiang and Lee (2007) for a discussion.
[7] See, Boudry, Kallberg, and Liu (2010).
[8] See, for example, Ott, Riddiough and Yi (2005), and Heinkel and Zechner (1990).
[9] See, for example, Bildersee (1973), Sorensen and Hawkins (1981), Emanuel (1983), and Ferreira, Spivey, and Edwards (1992).
[10] According to NAREIT (http://www.reit.com/investing/industry-data-research/reit-capital-offerings,) from 1992 through 2012 equity REITs raised $52.4 billion through preferred issuance, a number larger than the $41.3 billion raised in REIT IPOs, and approximately one fifth of all the public equity issued by REITs.

through preferred issuance, this asset class needs to offer attractive risk-return characteristics for investor portfolio formation and diversification.

To make our results as general as possible, we give our investor access to global and US large cap stock indices, US small and mid-cap equity indices, US small and mid-cap growth and value indices, investment grade and high yield bond indices, an equity REIT index, and a REIT preferred stock index. We conduct our analysis using data from the largest time series available, with our data spanning the period November 1992 to November 2012.[11]

We find three main results. First, both REIT common and preferred stock provide diversification benefits, but they provide these benefits to different sets of investors. In our utility based framework, under realistic constraints investors separate based on risk aversion. Investors with different levels of risk aversion will hold materially different portfolios. REIT common stock allows investors to form portfolios with higher total return. This is beneficial to low risk aversion investors who seek to form such portfolios. On the other hand, REIT preferred stock is a venue for risk reduction. This is prized by high risk aversion investors who allocate heavily to preferred stock. This view of REIT preferred and common stock is quite different to the one that is obtained using the classical unconstrained mean-variance framework. In that setting REITs provide little benefit. Adding realistic constraints to the investor's optimization problem materially changes how we should judge the benefits of the REIT sector to investors.

Second, REIT preferred stocks play an important role in portfolio formation. While the common perception is that preferred stock is a hybrid between common stock and investment grade bonds, our analysis indicates that preferred stock is actually preferred to a combination of those asset classes. We show that when an investor is given access to REIT preferred stock, REIT preferred stock displaces REIT common stock and investment grade bonds in their optimal portfolio allocation. This helps explains why REITs choose to issue these securities: there is a set of investors out there who prefer REIT preferred stock over holding a combination of REIT common stock and investment grade bonds.

Finally, we provide further evidence that examining diversification benefits using a constrained framework, results in materially different conclusions to those obtained from the traditional unconstrained framework.[12] Not only do constraints matter from a theoretical perspective, they can have a marked impact on the empirical conclusions drawn from the analysis of the data.

Any study of diversification is subject to the sample period used in the analysis. To examine the robustness of our results, we perform Monte Carlo simulations. Our Monte Carlo study shows that our results are not confined only to the historic time series examined. While the mean allocations to both REIT common and preferred stocks are lower in the Monte Carlo experiments, they are still quite large, with allocations of between 4.6 and 41.6% for REIT common stock and 14.0 and 30.7% for REIT preferred stock depending on the investor's level of risk aversion. The results of the Monte Carlo experiments fully support the conclusions from the analysis of the historical data. Once again, REIT common stock is beneficial to low risk aversion investors, while REIT preferred stock is more beneficial to higher risk aversion investors. Notice that while the portfolio allocations may seem extreme at say 41.6% of a portfolio in REIT common stock, this has to be interpreted in

[11] This sample period covers the modern REIT era. While REITs have existed since the 1960s, our sample period is the most relevant for a current REIT investor. Given that our methodology relies on return dynamics and there are large structural differences between the old and modern REIT eras, it is sensible to confine our interest to the latter time period.
[12] See Frost and Savarino (1988) and Jagannathan and Ma (2003) for a discussion of the benefits of including constraints in optimal portfolio decisions.

the context of our framework. In the classic mean-variance world, that would mean all investors hold such an extreme market portfolio. In our framework where investors segment based on risk aversion, it simply means that there is a subset of investors who would hold such an extreme portfolio. To some extent, this result is consistent with the extreme portfolios held by individuals that are documented in Barber and Odean (2000) and Goetzmann and Kumar (2008).

Overall our results are relevant to both academics and practitioners. First, they suggest that investors need to get away from judging diversification benefits simply using Sharpe Ratios and correlations. Investors face constraints and these constraints materially change how we should judge the benefits of asset classes. This is especially true for the REIT sector, because based on unconstrained Sharpe Ratios, the REIT sector provides no real benefits to investors. Once we impose realistic constraints, we observe the benefits that these securities provide. Second, our results also suggest that investors, especially those with moderate to high risk aversion who focus on equity portfolios, would be well served by investing in REIT preferred stocks. Doing so would be beneficial to the risk adjusted performance of their portfolios.

The remainder of this paper is organized as follows. Section 2 provides a review of the literature. Section 3 describes our empirical methodology and data. Section 4 presents our results, and Section 5 concludes.

2 Literature Review

The literature that this paper is most closely related to is the literature examining the role of real estate in mixed asset portfolios. This literature applies the Modern Portfolio Theory framework of Markowitz (1952) and Merton (1972) to examine the benefit of holding real estate (both direct and securitized in the form of REITs) in mixed asset portfolios.

Beginning in the early 1980's a series of papers demonstrates that positive allocations to real estate produce superior risk-adjusted returns to portfolios that omit real estate, especially for risk sensitive investors. These studies rely on the NCREIF index of real estate returns, a database of private real estate returns, or an index of returns from comingled real estate funds. Using these data, Miles and McCue (1984) find that real estate acts as an inflation hedge and that there are other intriguing properties of real estate returns, notably the low correlation with stocks and bonds; Ross and Zissler (1991) estimate allocations in the range of 9 to 13 percent while Folger (1984) makes the case for a 20% allocation using these data. Ibbotson and Siegel (1984) confirm the findings of Miles and McCue (1984) and Folger (1984), but also show that the pricing of real estate reacts to very different stimuli than stocks and bonds; real estate prices are shown to be sensitive to residual risk and factors such as taxes, marketability and information costs that are not captured in the traditional measure of risk as represented by market or beta risk.

An early critique of these studies is that they rely on appraisal based returns indexes, which are known to understate the volatility of real estate returns, called appraisal smoothing. Corgel and deRoos (1999) correct for the understated volatility and find significant allocations to private real estate in the ranges found by previous authors. While the correction for appraisal smoothing adds volatility to real estate returns, thus reducing the allocation, the correction also makes real estate less correlated with stocks and bonds, increasing the allocation, even when REITs are available as an investment. Kallberg, Liu, and Greig (1996) find support for allocations of approximately 9% to real estate, but in addition, find a "size effect," that is, small properties tend to bring much larger diversification benefits to mixed asset portfolios than large properties due to their lower correlation with other financial assets while maintaining a similar risk/return profile as large properties.

Starting in the mid-1990's, real estate researchers added REITs to the NCREIF return series, allowing an exploration of both public market real estate returns in addition to the private market returns. Gyourko and Nelling (1996) explore the impact of different property types and location in real estate portfolio allocation and find that diversification by location and property type is independent to market based diversification measures. Kallberg, Liu, and Trzcinka (2000) find that REIT mutual funds have persistent positive net alphas, a significant finding. Combining the conventional wisdom that real estate provides diversification and an inflation hedge, Lin and Yung (2006) find that REIT returns affect REIT equity market flows, not vice versa, meaning that the market demand for REIT equities is closer to horizontal than downward sloping.

Some papers have questioned the benefits of REITs in mixed asset portfolios. Most notably, Chiang and Lee (2007) demonstrate using spanning tests that the diversification benefits of REIT common stock is very sensitive to the set of benchmark assets. In many cases REIT common stock is spanned by the test assets and as such provides no diversification benefits.

More recently, Sa-Aadu, Shilling, and Tiwari (2010) examine the performance of real estate in mixed asset portfolios in a conditional asset pricing framework and find that real estate is one of two asset classes that deliver portfolio gains during bad times; that is, when investors really care about returns. They argue that investors may underweight real estate due to this misunderstanding of real estate return behavior in bad times.

3 Data and Methodology

3.1 Methodology

In the standard mean-variance setting where investors are unconstrained in their portfolio formation and can long and short the risk free asset, all investors hold portfolios that are a combination of the risk free asset and the market portfolio (two fund separation). The composition of the market portfolio (relative weights of different risky securities) is independent of investor preferences. In this setting diversification benefits provided by a given risky asset can be measured by a Sharpe ratio and the improvement in the Sharpe ratio is independent of investor risk preferences. That is, if an asset provides significant diversification benefits, the benefits can be evaluated without a discussion of investor preferences. However, when investors face short sales constraints and cannot borrow at the risk free rate there are important changes. Two fund separation no longer holds and the way we need to think about diversification benefits provided by a given risky asset also needs to change. Investors no longer hold risky assets in the same proportion, and their portfolio holdings depend on their risk preferences. We now develop this case in more detail.

To illustrate the role of short sales constraints, consider a simple constrained portfolio problem. Let there be two risky assets, with returns jointly normally distributed with expected returns $\mu_1,\ \mu_2$, standard deviations σ_1, σ_2, and correlation ρ.[13] Without loss of generality assume that $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$. Consider investors with negative exponential utility of wealth, $U(W) = -e^{-a_i W}$, where $a_i > 0$ is risk aversion for investor i. An investor is endowed with initial wealth, W_0, and maximizes expected value of terminal wealth. When there is no risk-free asset and no short sales of risky assets, an investor chooses investments in the two risky assets, w_1 and w_2, to maximize expected utility,

[13] The results in this section hold for the general case of n risky assets with expected returns vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The two asset case is described for tractability and is without loss of generality. Ross (1977) and Dybvig (1984) study the shape of efficient frontiers under constraints, but do not include investor preferences.

$$\max_{\substack{w_1, w_2 \\ w_1 \geq 0,\ w_2 \geq 0 \\ w_1 + w_2 = W_0}} E\left[U\left(\widetilde{W}\right)\right] = \max_{\substack{w_1, w_2 \\ w_1 \geq 0,\ w_2 \geq 0 \\ w_1 + w_2 = W_0}} -e^{-a_i(\mu_p - 0.5\, a_i\, \sigma_p^2)}$$

where μ_p and σ_p are the expected return and standard deviation of the portfolio with w_1 invested in asset one and w_2 invested in asset two.[14] The equality obtains because a linear combination of normally distributed variables (assets) is normally distributed (portfolio), and by taking the integral of negative exponential utility function with respect to the probability density function of a Normal distribution. To solve this maximization problem, we need to solve

$$\max_{\substack{w_1, w_2 \\ w_1 \geq 0,\ w_2 \geq 0 \\ w_1 + w_2 = W_0}} (\mu_p - 0.5\, a_i\, \sigma_p^2)$$

Let λ_1 and λ_2 be the Lagrange multipliers for the two inequality constraints, and φ the Lagrange multiplier associated with the budget constraint. Forming the Lagrangian and differentiating it with respect to w_1, w_2, the solution to the problem is:

1. Corner solution, with all wealth in the first risky asset: $w_1 = W_0, w_2 = 0, \lambda_1 = 0, \lambda_2 = \mu_1 - \mu_2 - a_i W_0(\sigma_1^2 - \rho\sigma_1\sigma_1), \varphi = 1 + \mu_1 - a_i W_0 \sigma_1^2$.

2. Corner solution, with all wealth invested in the second risky asset (symmetric to the first case above): $w_1 = 0, w_2 = W_0, \lambda_1 = -\mu_1 + \mu_2 - a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2), \lambda_2 = 0, \varphi = 1 + \mu_2 - a_i W_0 \sigma_2^2$.

3. Interior solution (both risky assets held in positive quantity): $\lambda_1 = 0, \lambda_2 = 0$,

$$w_1 = \frac{\mu_1 - \mu_2 + a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

$$w_2 = \frac{-\mu_1 + \mu_2 + a_i W_0(\sigma_1^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

$$\varphi = \frac{-(2 + \mu_1 + \mu_2)\rho\sigma_1\sigma_2 + (1 + \mu_1)\sigma_2^2 + (1 + \mu_2)\sigma_1^2 - a_i W_0 \sigma_1^2 \sigma_2^2 (1 - \rho^2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}$$

Consider the interior solution. What types of investors invest more in the first asset than in the second asset? That is, for what values of risk aversion a_i, is $w_1 > w_2$? Note that the denominator in the expressions for w_1 and w_2 is positive, because risk aversion is positive ($a_i > 0$) and $(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2) > 0$ because this is the variance of the portfolio that invests one in the first asset and minus one in the second asset. Then,

$$w_1 > w_2 \Leftrightarrow \mu_1 - \mu_2 + a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2) > -\mu_1 + \mu_2 + a_i W_0(\sigma_1^2 - \rho\sigma_1\sigma_2)$$

$$\Leftrightarrow a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2) - a_i W_0(\sigma_1^2 - \rho\sigma_1\sigma_2) > 2(\mu_2 - \mu_1)$$

[14] See Kroll, Levy, and Markowitz (1984) for a discussion of mean variance optimization versus utility maximization. Their results suggest that for a wide range of utility functions and return distributions, mean variance optimization is going to give close approximations to utility maximization. Given our assumptions of negative exponential utility and normal returns, all utility maximizing portfolios will be mean-variance optimal portfolios.

$$\Leftrightarrow a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2 - \sigma_1^2 + \rho\sigma_1\sigma_2) > 2(\mu_2 - \mu_1)$$

$$\Leftrightarrow a_i > \frac{2(\mu_2 - \mu_1)}{W_0(\sigma_2^2 - \sigma_1^2)}$$

Recall that the two assets are ordered so that $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$. Therefore, there exists sufficiently high level of risk aversion, so that investors with risk aversion above this level of risk aversion hold more wealth in the less risky asset, than in the more risky asset.

This analysis shows that in the presence of short sales constraints and no risk free asset, investors no longer hold risky securities in the same proportion. It also shows that investor risk aversion affects the composition of the portfolio. Because different investors hold risky assets in different proportions, diversification benefits of a given risky asset will be different for different investors. Notice that the inability to short sell the risky assets is critical. If investors could short sell the risky assets, then two fund separation would be possible once again. As Black (1972) shows, in this case the investor holds a portfolio that is a combination of the market portfolio and the minimum variance zero beta portfolio. Below we show that this break down of two fund separation continues to occur when a risk-free asset is a part of the investment opportunity set, but investors cannot borrow at the risk free rate.

3.1.1 Constrained investment in the riskless asset

Now consider a case where a risk free asset paying the rate of interest r is a part of the investment opportunity set, but investors cannot borrow at a risk-free rate ($w_{rf} \geq 0$).[15] The utility maximization problem is now

$$\max_{\substack{w_1, w_2, w_{rf} \\ w_1 \geq 0,\ w_2 \geq 0,\ w_{rf} \geq 0 \\ w_1 + w_2 + w_{rf} = W_0}} E\left[U\left(\widetilde{W}\right)\right] = \max_{\substack{w_1, w_2,\ w_{rf} \\ w_1 \geq 0,\ w_2 \geq 0,\ w_{rf} \geq 0 \\ w_1 + w_2 + w_{rf} = W_0}} -e^{-a_i(\mu_p - 0.5\, a_i\, \sigma_p^2)}$$

Form the Lagrangian,

$$\begin{aligned} L&\left(w_1, w_2, w_{rf}, \lambda_1, \lambda_2, \lambda_3, \varphi\right) \\ &= (1+\mu_1)w_1 + (1+\mu_2)w_2 + (1+r)w_{rf} - \frac{1}{2}a_i(w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2) \\ &- \lambda_1(-w_1 - 0) - \lambda_2(-w_2 - 0) - \lambda_3\left(-w_{rf} - 0\right) - \varphi\left(w_1 + w_2 + w_{rf} - W_0\right) \end{aligned}$$

Differentiating the Lagrangian and equating the derivatives to zero, we obtain characterization of the solution (detailed characterization of the solution is given in the Appendix). The solution consists of seven cases: (1) Interior solution, positive investment in the risk-free asset and both risky assets. (2) Corner solution: All money is invested in the risk-free asset. (3) No investment in the first risky asset, positive investment in the second risky asset and risk-free asset. (4) Corner solution: all funds invested in the second risky asset. (5) No investment in the second risky asset, positive investment in the first risky asset and risk-free asset. (6) Corner solution, all funds invested in the first risky asset. (7) Portfolios with zero investment in the risk free asset and non-negative investment in the risky assets.

[15] This is one of the cases considered by Ross (1977). He shows that under this set of assumptions, the CAPM breaks down. Given that his focus was on situations in which the CAPM fails to price all assets, he does not explore the implications of these constraints on optimal portfolio holdings and how these are affected by risk aversion.

It is shown in the Appendix that cases (2), (3), (5), and (6) are dominated by case (1). The important cases are: (1) portfolios with investment in the risk free asset and both risky assets; (7) portfolios with no investment in the risk free asset and positive investment in the two risky assets; and (4) all funds invested in the second risky asset. We now discuss these cases.

Case 1. Interior Solution: Positive investment in the risk-free rate and both risky assets:

$$w_{rf} = \frac{\mu_2\sigma_1(\sigma_1 - \rho\sigma_2) - r(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2) + a_i W_0(\rho^2 - 1)\sigma_1^2\sigma_2^2 + \mu_1\sigma_2(\sigma_2 - \rho\sigma_1)}{a(\rho^2 - 1)\sigma_1^2\sigma_2^2}$$

$$w_1 = \frac{(\mu_1 - r)\sigma_2 - (\mu_2 - r)\rho\sigma_1}{a_i\, \sigma_1^2\sigma_2(1 - \rho^2)}, \qquad w_2 = \frac{(\mu_2 - r)\sigma_1 - (\mu_1 - r)\rho\sigma_2}{a_i\, \sigma_1^2\sigma_2(1 - \rho^2)}$$

The tangency portfolio (a straight line with the origin at the risk-free rate and tangent to the efficient frontier curve) is characterized by zero investment in the risk free asset. The weights of the two risky assets in the tangency portfolio are:

$$w_{1,Tangency} = W_0 \frac{\big(\mu_2\rho\sigma_1 - \mu_1\sigma_2 + r(\sigma_2 - \rho\sigma_1)\big)\sigma_2}{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho\sigma_1\sigma_2 + (r - \mu_1)\sigma_2^2}$$

$$w_{2,Tangency} = W_0 \frac{\big(-\mu_2\sigma_1 + \rho\mu_1\sigma_2 + r(\sigma_1 - \rho\sigma_2)\big)\sigma_1}{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho\sigma_1\sigma_2 + (r - \mu_1)\sigma_2^2}$$

As investor risk aversion increases, the investor holds more in the risk free asset. As risk aversion decreases, investment in the tangency portfolio increases. This case is the closest to the canonical unconstrained portfolio problem where two fund separation obtains. An important distinction, however, exists. In the canonical unconstrained problem the two fund separation covers *all* optimal portfolios and therefore applies to *all* investors. In the constrained case we consider, this is only part of the efficient frontier, and it applies only to investors with sufficiently high risk aversion.

Case 7. Interior Solution: Zero investment in the risk free asset and positive investment in the risky assets,

$$w_1 = \frac{\mu_1 - \mu_2 + a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

$$w_2 = \frac{-\mu_1 + \mu_2 + a_i W_0(\sigma_1^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

The composition (relative investment in the risky assets) of the portfolios in this segment depends on investor risk aversion, a_i. As risk aversion decreases, a higher proportion of funds are invested in the riskier asset.

Case 4. This is simply a corner solution with all funds invested in the risky asset with the highest expected return (the second risky asset).

The efficient frontier in the standard deviation (x-axis) - return (y-axis) space is shown in Figure 1, and characterized as follows (the derivation is in the Appendix):

Segment 1 - Straight line with the origin at the risk-free rate, ending at the tangency portfolio. Portfolios on this segment are characterized by the case 1 and include investment in the risk-free asset and in the tangency portfolio. Investors with sufficiently high risk aversion hold portfolios on this segment,

$$a_i > \frac{(r-\mu_2)\sigma_1^2 + (\mu_1+\mu_2-2r)\rho\sigma_1\sigma_2 + (r-\mu_1)\sigma_2^2}{W_0(-1+\rho^2)\sigma_1^2\sigma_2^2}$$

Segment 2 - A curve beginning at the tangency portfolio and ending at the risky asset with the highest expected return (the second risky asset). Investors whose risk aversion satisfies the inequality below hold portfolios located in this segment,

$$\frac{\mu_2-\mu_1}{W_0(\sigma_2-\rho\sigma_1)\sigma_2} < a_i \leq \frac{(r-\mu_2)\sigma_1^2 + (\mu_1+\mu_2-2r)\rho\sigma_1\sigma_2 + (r-\mu_1)\sigma_2^2}{W_0(-1+\rho^2)\sigma_1^2\sigma_2^2}.$$

Segment 3 - Corner solution, with all wealth invested in the second risky asset. Investors with sufficiently low risk aversion hold this allocation,

$$0 < a_i \leq \frac{\mu_2-\mu_1}{W_0(\sigma_2-\rho\sigma_1)\sigma_2}.$$

What is crucial to observe is that the composition of the optimal portfolio along the overall frontier (the relative amount invested in the risky assets) depends on investor risk preferences. Investors with sufficiently high risk aversion will hold portfolios on segment 1; investors with lower risk aversion will hold portfolios on segment 2, with all portfolios on this segment having different composition of the risky assets; and finally investors with sufficiently low risk aversion will hold a corner solution (segment 3).[16]

In this setting, different segments of the frontier are composed of different assets, based on their return characteristics (means, variances, and correlations.) Because risk aversion separates investors across segments of the frontier, diversification benefits will be different for investors with different risk preferences. This is our main theoretical conclusion. It implies that investor characteristics must be taken into account when measuring and discussing diversification benefits of various asset classes in the investment opportunities set.

3.1.2 A Utility-Based Framework for Measuring Diversification Benefits: Compensation Ratios

When investors face short sales constraints and can't borrow at the risk free rate, the relative compositions of their optimal portfolios depend on risk preferences. Diversification benefits of a risky asset, then, need to be measured taking into account investor preferences. In other words, a utility-based framework for measuring diversification benefits is needed. To assess diversification benefits of different assets to different investors we employ compensation ratios.

Consider an investor with initial wealth W_0 solving an expected utility maximization problem with a risk free asset (no borrowing), and n risky securities (the unrestricted set). The investor achieves derived utility of wealth,

$$V_n(W_0) = E\left[U\left(\widetilde{W}\right)\right],$$

[16] Notice that the constraint on both the risky assets and the risk free rate are important. If the investor could borrow at the risk free rate, then two fund separation would hold even in the face of short sales constraints on the risky assets (see Ross (1977).)

where the subscript n denotes that investor had access to unrestricted asset set with n risky assets. Now, restrict the investor from investing in one of the risky assets, namely the asset whose diversification benefits are being assessed.[18] The investment opportunity set now contains $(n-1)$ assets and the investor achieves derived utility of wealth under the constrained set (denoted by the subscript $(n-1)$),

$$V_{n-1}(W_0) = E\left[U\left(\widetilde{W}\right)\right].$$

Wealth compensation, ΔW_k is the additional wealth required when asset k is removed from the investment opportunity set to restore utility to the level achieved under the unrestricted risky asset set, and is the solution to the equation,

$$V_{n-1}(W_0 + \Delta W_k) = V_n(W_0).$$

Wealth compensation is measured in terms of time zero (initial) wealth. To evaluate the impact of different assets in the investment opportunity set, consider *compensation ratio* -- a ratio of compensation required for removal of asset k relative to compensation required for removal of asset j,

$$Compensation\ Ratio = \frac{\Delta W_k}{\Delta W_j}.$$

This approach can be applied to individual assets, as well as to collections of assets. For example, we examine compensation ratios (as a function of risk aversion) of the wealth required to compensate an investor for losing access to REIT preferred stock (or REIT common stock) dividend by the compensation required to compensate them for losing access to both REIT common and preferred stock. In this sense the ratios measure the relative importance of REIT preferred stock and REIT common stock to an investor over different levels of risk aversion.

3.2 Data

We collect monthly returns for 13 indices from November 1992 to November 2012 from four sources. Datastream provides returns on the Barclays Investment Grade Corporate Bond Index, the Barclays High Yield Corporate Bond Index,[19] the MSCI World Ex-US index, the Russell 2000 index, the Russell 2000 growth index, the Russell 2000 value index, the Russell Mid Cap index, the Russell Mid Cap growth index, and the Russell Mid Cap value index. From SNL we collect the SNL US Equity REIT index. MSCI provides the MSCI REIT Preferred Index. From CRSP we obtain returns on the 30-day T-Bill and the returns on the S&P500.[20]

All of the data we use in the analysis is quite standard apart from the MSCI REIT Preferred Index, so some discussion of that series is warranted. REITs are one of the few industries that issue large quantities of preferred stock. According to NAREIT, from 1992 through 2012 equity REITs raised \$52.4 billion through preferred issuance, a number larger than the \$41.3 billion raised in REIT IPOs, and approximately one fifth of all the

[18] Ukhov (2006) studies changes in the frontier caused by addition and removal of assets from the opportunity set.
[19] It would be interesting to include a REIT specific bond index in the analysis, however we are unaware of any such index being available. Furthermore, an examination of the TRACE database indicates that constructing such an index might be highly problematic due to the lack of liquidity in the REIT corporate bond market.
[20] The one asset class we do exclude from the analysis is private real estate. The reason for this is that it isn't apparent that an investor can easily obtain these returns. NCREIF appraisal based returns have well known statistical issues that would be particularly problematic in a mean-variance setting and Boudry, Coulson, Kallberg and Liu (2013) show that transactions-based returns can't easily be replicated by investors.

public equity issued by REITs.[21] The typical REIT preferred stock has a fixed dividend rate, is issued at par, and is callable after 5 years. In terms of dividend priority, preferred stock dividends take priority in payment to common dividends, and also typically count towards the dividend payout requirements of the REIT.[22] The MSCI REIT Preferred Index consists of non-convertible preferred stock issued by public US equity and hybrid REITs, with the further requirement that the preferred issue be traded on the NYSE, AMEX, or NASDAQ. [23]

Table 1 reports descriptive statistics for the indices used in our analysis. Panel A reports means and standard deviations, while Panel B reports correlations. The average annualized return for REIT preferred stock over the sample period was 10.3% with a standard deviation of 11.4%. This compares to an average return of 12.9% for REIT common stock, which was the asset class with the highest average return in our sample. The S&P 500 also performed well over the sample period with a mean return of 9.2%. As would be expected over a long sample, investment grade bonds had the lowest average return of 7.1%.

Turning to the correlations in Panel B, we observe some noticeable characteristics. First, preferred stock is most highly correlated with high yield bonds with a 68% correlation, but also shares significant co-movement with REITs with a 62% correlation. Second, an examination of the correlation between REIT common stock and the other equity indices shows that REITs are more highly correlated with small and mid-cap stocks than large cap, but also that REITs appear to have a significant value component to them. For both the mid-cap and Russell 2000 universes, REITs are more highly correlated with value stocks than growth stocks.[24] Finally, the asset class that appears to be least correlated with the others is investment grade bonds. It has high correlations with only the high yield bond index and REIT preferred stock index.

Figure 2 reports the current value of a dollar invested in each asset class in 1992. Some clear patterns emerge regarding the time series behavior of the asset classes. First, consistent with the average returns observed in Table 1, REIT common stock was the best performing asset class by the end of the sample period. Notice however, that this was not always the case. Prior to 2000 REIT common shares were nearly always the worst performing equity index and up to that time had a total return nearly identical to high yield corporate bonds. In fact prior to 2002, REIT common stock and preferred stock had remarkably similar performance. Second, from 2002 to 2007 we observe a marked increase in the performance of midcap, small and midcap value, and

[21] Lee and Johnson (2009) report that 20% of NYSE firms, 15% of AMEX firms and 17% of NASDAQ firms have preferred stock in their capital structure; Yaman (2011) reports that that slightly more than one-third of all non-IPO equity raised by U.S. firms was preferred equity over the 1985-1999 timeframe, with the U.S. public market for preferred stock reaching $193billion in 2005. Kallberg, Liu and Villupuran (2013) report that U.S. firms used preferred equity to raise over 47% of all equity capital from 1999 – 2005, including IPOs; further preferred equities were almost 60% of all non-IPO equity raised during this period.

[22] See Boudry (2011) for a discussion of the dividend payout requirements of REITs.

[23] We are able to obtain the constituents of the MSCI REIT Preferred Index from 2005 on. An examination of these constituents shows that the index is extremely broad based. There are over 100 constituents in the index at any time and no individual constituent is more than 2% of the index. We are also able to obtain daily REIT preferred stock prices and trading volume from 2000 on and observe that most REIT preferred stock trades at a daily frequency. This suggests that although they are less liquid than there common share counterparts, they are liquid enough for an investor to be able to form portfolios.

[24] Notice that this is unlikely to be driven by REITs being components of midcap and small cap indices for two reasons. First, these are very broad based indices. Second, the Russell 2000 value index has a 97% correlation with the small/value Fama French portfolio that by construction does not include REITs. The REIT common index also has a correlation of 73% with the small/value portfolio. In this sense it appears that REITs are highly correlated with small and midcap value indices not because they are part of those indices, but because they tend to be small and midcap value stocks. We use the Russell indices in our analysis instead of Fama French portfolios simply because an investor is far more likely to hold a Russell index than a portfolio of stocks they created with size and book to market sorts.

REIT common shares. The close relationship between REIT common shares and midcap value stocks become quite apparent in Figure 3 when we plot just the REIT common and preferred shares, the S&P 500, and the mid cap value index. Finally, the effects of the global financial crisis, subsequent recession, and recovery are evident in the latter part of the sample. We observe all risky asset classes declining significantly during the crisis and rebounding during the recovery. This V-shaped pattern being quite pronounced in both the REIT common and preferred shares.

4 Results

4.1 Classical Analysis

We begin our minimum-variance analysis using the standard mean-variance framework of Markowitz (1952) where the investors are unconstrained in their portfolio formation and can freely borrow and lend at the risk free rate. In such a framework, the diversification benefits of any given asset can be measured by its improvement in the Sharpe Ratio of the tangency portfolio. Table 2 shows the characteristics of the market (tangency) portfolios created with different mixes of equity and debt asset classes. The left columns are for the case where bonds are excluded from the analysis and the right columns refer to when bonds are included in the investor's opportunity set. Panel A is the most inclusive opportunity set and includes all equity classes. Panel B removes the value and growth portfolios but leaves the investor access to size portfolios. Panel C gives the investor access to size and value/growth portfolios. Panel D is the most restrictive set and give the investor access only to the S&P 500 and the World-Ex-US portfolios. In each case we exclude both REIT common and preferred stock in the base case and then compare improvements in the Sharpe Ratio when either REIT preferred stock, or REIT common, or both is added to the opportunity set.

A few results are apparent from Table 2. REIT common stock is really only valuable when the investor is given a very limited opportunity set. This is evident in the left columns of Panel D where the Sharpe Ratio improves by 28.62% (from 0.1202 to 0.1546, 0.4164 to 0.5418 annualized)[25] when the investor is given access to REIT common stock. However, notice that this effect is mitigated if the investor is given access to bonds (Panel D right columns) or value/growth portfolios (Panel C). In this sense REIT common stock appears to be spanned by a value/growth factor and a fixed income factor, at least in this standard setting. This is important, because numerous studies do not give investors access to debt or value/growth portfolios. It is also supportive of the results of Chiang and Lee (2007) who find that the diversification benefits of REIT common stock is sensitive to benchmark portfolios.

The second result from Table 2 is that REIT preferred stock appears to behave very much like a bond substitute. Investors who are constrained to invest only in equity asset classes always find access to preferred stock beneficial.[26] The magnitude of the improvement ranges from 8.13% when all other equity asset classes are included (Panel A left columns), to 63.39% when the most restrictive set of equity classes is considered (Panel D left columns). In fact nearly all the benefit to equity investors from the REIT market comes from access to their preferred stock, not their common stock (compare the first and third row in each panel.). Once investors have access to bond indices, the benefits of preferred stock are reduced quite dramatically. This is

[25] To obtain annualized returns, monthly returns should be multiplied by 12; to obtain annualized standard deviation, monthly standard deviation should be multiplied by square root of 12. Sharpe ratio is defined as $(r - r_f)/\sigma$. Therefore, the annualized Sharpe Ratio is obtained from Sharpe Ratio based on monthly numbers by multiplying it by $\sqrt{12}$.

[26] The economic significance is similar in magnitude to the benefits from diversifying internationally by UK equity investors as reported in Goetzmann and Ukhov (2006).

evident from the reduced increases in Sharpe Ratios reported in the right columns of Table 2. In the most inclusive case, REIT preferred stock improves the Sharpe Ratio by only 0.24%.[27]

Taken together the results of Table 2 suggest that REITs potentially provide little diversification benefit to investors, when benefits are measured within the classical unconstrained portfolio problem. If an investor is given access to easily tradable portfolios of value stocks or bonds, then Sharpe Ratios are not improved greatly by also giving the investor access to REIT common or preferred stock. The unconstrained analysis thus leads one to question why investors should hold REITs in their portfolios.

4.2 Constrained Analysis

We start our constrained analysis by considering the simplest case where there is no risk free asset and the investor cannot short the risky assets. While this scenario may be extreme, it provides a base for understanding how constraints affect investors and is easily extendable to more realistic cases.

4.2.1 No Risk Free Asset and No Bonds

Figure 4 reports efficient frontiers and indifference curves for the case of a constrained risky asset portfolio (no shorts) and no risk free asset. In Panel A, the dashed lines represent the case where preferred stock is excluded from the opportunity set and the solid lines represent the case where they are included. Black lines are the mean-variance frontier, while red lines are indifference curves for the high risk aversion investor and blue lines are indifference curves for the low risk aversion investor. In Panel B we repeat the exercise, but dashed lines represent the case where REIT common stock is excluded from the opportunity set and the solid lines represent the case where they are included. In both cases the investment opportunity set includes all the other non-REIT equity classes.

Examining Panel A, we observe that the addition of preferred stock has a dramatic effect on the mean-variance frontier. Preferred stock provides a venue for significant risk reduction. This is evident in the shift to the left of the frontier from the dashed curve to the solid curve. This risk reduction is most beneficial to the high risk aversion investor. The high risk aversion investor's tangency portfolio changes quite dramatically and they are able to obtain a higher indifference curve (solid red line lies above the dashed red line). However, notice that the addition of preferred stock does little for the low risk aversion investor. Their tangency portfolio remains essentially identical (solid blue and dashed light blue lines lie on top of each other.) This is the sense in which one needs to be specific about which set of investors benefit from access to a given asset. Preferred stock is highly beneficial to high risk aversion investors, but of little consequence to low risk aversion investors.

Turning to Panel B, we observe that the addition of REIT common stock has a different effect on the mean-variance frontier. Including REIT common stock allows investors to form portfolios with higher total return than they otherwise would have been able to attain. That is, the solid efficient frontier (including REIT common) lies above the dashed efficient frontier curve (excluding REIT common) with this difference being most pronounced for high return portfolios. In this case it is the low risk aversion investor who benefits. Their optimal portfolio changes and they are able to obtain a higher indifference curve when given access to REIT common stock (solid blue indifference curve is above the dashed light blue indifference curve.) Notice that this is true even though the investor already has access to both size and value/growth tilted portfolios. Examining

[27] This is consistent with the thesis put forward by Graham and Dodd (1934) that preferred stock is simply a hybrid security spanned by debt and equity.

the high risk aversion investor, we see that they are largely indifferent to access to REIT common stock (the solid red and dashed red indifference curves lie on top of each other.) This once again demonstrates how an asset class can have different benefits to different investors. REIT common stock benefits low risk aversion investors, but is of little consequence to high risk aversion investors.

To further examine the benefit of REIT common and preferred stock to investors with different risk aversions, in Figure 5 we plot compensation ratios for investors with different levels of risk aversion. The compensation ratio is measured as the wealth required to compensate an investor for losing access to preferred stock (blue line) or REIT common (red line) divided by the compensation required to compensate them for losing access to both REIT common and preferred stock. In this sense they measure the relative importance preferred stock (blue line) and REIT common (red line) to an investor over different levels of risk aversion.

Both the blue and red lines confirm what we observed in Figure 4 for only two levels of risk aversion. The benefit of access to REITs both in the form of common and preferred stock is dominated by preferred stock for high risk aversion investors (red line declines with risk aversion.) Examining the blue line (that excludes preferred stock), for high levels of risk aversion the compensation ratio moves towards one. This indicates that for high risk aversion investors, nearly all the compensation they require for losing access to the REIT market is because of the lack of access to REIT preferred stock. If we examine the red line (no REIT common) we see the opposite is true for low risk aversion investors. Most of the compensation for losing access to the REIT assets is due to losing access to REIT common stock.

To provide some economic significance to our analysis, in Figure 6 we plot the weights in the optimal portfolio of REIT preferred stock (blue), REIT common stock (red), and all other equities (green) for investors of different risk aversion. In both Panel A and Panel B, the heavy solid line represents the base case for the analysis where the investor has access to all the equity asset classes including REIT common and preferred stock. For extremely low levels of risk aversion, we observe that the investor's optimal portfolio is dominated by REIT common stock. This allocation declines as risk aversion increases, although is non-zero for many levels of risk aversion that would be considered plausible in the literature.[28] Turning our attention to preferred stock, we observe that a low risk aversion investor puts very little weight in preferred stock, but this allocation grows quite markedly as risk aversion increases. For high levels of risk aversion we observe that the investor's portfolio is dominated by preferred stock. While such an allocation is economically unreasonable for what we observe in practice, in this scenario the investor doesn't have access to either risky or riskless fixed income securities.

The dashed lines in Panel A of Figure 6 represent the case where the investor is not given access to REIT common stock and we examine how the investor allocates their portfolio over different levels of risk aversion. At low levels of risk aversion, the investor's allocation to preferred stock is slightly increased compared to the case with all equity asset classes. The gap left by not having access to REIT common stock is filled by increasing the allocation to the other non-preferred equity classes. This suggests that REIT preferred stock is not a great substitute for REIT common stock.

In Panel B we repeat the analysis but in this case the dashed line represents the case where the investor doesn't have access to REIT preferred stock. In this case we observe that while the allocation to other equity classes increases for all levels of risk aversion, so does the allocation to REIT common stock, especially for higher risk

[28] See Bliss and Panigirtzoglou (2004), Blackburn, Goetzmann, and Ukhov (2009), and Edelstein and Magin (2013) for a discussion.

aversion investors. This suggests that REIT common may provide some bond like characteristics that the risk averse investor seeks, but cannot obtain in other equity classes.

4.2.2 Long Only Risk Free Asset and No Bonds

We now extend our analysis by giving the investor access to a risk free security. Importantly, we allow the investor to freely invest, but not borrow at the risk free rate. This matches the realistic constraint that investors' face in practice. In Figure 7 we plot the mean-variance frontier and indifference curves as in Figure 4. In Panel A, the solid line refers to the case with REIT preferred stock and the dashed the case without REIT preferred stock. In Panel B, the solid line refers to the case with REIT common stock and the dashed the case without REIT common stock. Red indifference curves relate to the high risk aversion investor, while blue indifference curves are for the low risk aversion investor. Because we have introduced the risk free asset, the investor is no longer constrained to portfolios that sit on the mean-variance frontier. The mean-variance efficient portfolios (black lines) are made up of the line starting from the risk free rate and extending to the green tangency portfolio. The efficient frontier then traces along the upper portion of the mean-variance curve.

Examining Figure 7 Panel A, we can determine the usefulness of preferred stock to investors. Once again the inclusion of preferred stock shifts the mean-variance frontier to the left. This allows high risk aversion investors to attain a higher indifference curve (solid red indifference curve lies above the dashed red indifference curve.) As was the case without the risk free rate, access to the preferred stock has little effect on the low risk aversion investor. Their portfolio and level of utility remain unchanged (solid blue and dashed blue lie on top of each other.)

Turning to Figure 7 Panel B, we examine the effect of giving the investor access to REIT common stock. As was the case without the risk free asset, including REIT common stock allows investors to form portfolios with higher total returns (the solid efficient frontier lies above the dashed frontier curve.) This is once again beneficial to the low risk aversion investor. They are able to attain a higher indifference curve when they have access to REIT common stock (solid blue indifference curve is above the dashed light blue indifference curve.) The high risk aversion investor is unaffected by the inclusion of REIT common stock. They hold the same portfolio and attain the same level of utility they did when they did not have access to REIT common stock.

Taken together, the results of Figure 7 show that giving an equity investor access to invest in the risk free rate has not changed their preferences towards REIT common and preferred stock. High risk aversion investors still benefit from access to REIT preferred stock, and low risk aversion investors still benefit from access to REIT common stock.

Not surprisingly, when we examine the compensation ratios in Figure 8, we observe a similar relationship to the one seen in Figure 5. High risk aversion investors still require the majority of their compensation for losing access to REIT preferred stock, while low risk aversion investors require relatively more compensation for losing access to REIT common stock.

Examining the optimal portfolio weights in Figure 9, we observe that both REIT common and preferred stock still form an economically significant portion of investor portfolios. In Figure 9, the red line represents the weight in REIT common stock, the blue line is the weight in REIT preferred stock, the green line is the weight in the other equity asset classes, and the black line is the allocation to the risk free asset. Panel A is for the case where the investor has access to all equity asset classes, while Panel B is the case where the investor no longer has access to REIT common stock, and Panel C is the case where the investor no longer has access to REIT preferred stock.

Contrasting Figure 9 Panel A to Figure 6, we observe the main effect of introducing the risk free rate is to dampen the allocation to preferred stock for high levels of risk aversion. Highly risk averse investors substitute out of REIT preferred stock and into the risk free asset, while allocations to REIT common stock and the other equity classes remain similar to before. This makes intuitive sense because as the investor's risk aversion increases they are slowly holding a portfolio that is dominated by the risk free asset.

If we remove the investor's ability to invest in REIT common stock (Panel B) we observe, as before, that the investor allocates slightly more to REIT preferred stock at lower levels of risk aversion and substantially more the other risky asset classes. If we remove the investor's access to REIT preferred stock (Panel C) we observe that the investor allocates more heavily towards REIT common stock than they did before, but now with access to the risk free asset, they also start investing in the risk free asset at lower levels of risk aversion. They also invest substantially more in the risk free asset at extremely high levels of risk aversion. In this sense the investor appears to be attempting to replicate REIT preferred stock as a combination of REIT common stock and the risk free asset.

4.2.3 Long Only Risk Free Asset Plus Bonds

The final case we consider is when the investor has access to all the equity asset classes, can invest in the risk free asset, and also has access to investment grade and high yield debt. This represents the most realistic investment opportunity set that an investor might have. And remember from Table 2, it is also the case where in the classical unconstrained Markowitz setting, REIT common and preferred stock appeared to provide little if any benefit to investors.

Figure 10 reports mean-variance frontiers and indifference curves for the case with investment grade and high yield bonds, investment in the risk free asset, and access to all the equity securities. The high risk aversion investor's indifference curves are in red, while the low risk aversion investor's indifference curves are in blue. In Panel A, the solid lines are for the case where the investor has access to all securities, while the dashed case is when the investor does not have access to REIT preferred stock. In Panel B, the dashed lines refer to the case where the investor does not have access to REIT common stock. As was the case in Figure 7, efficient portfolios are those that extend from the risk free rate to the green tangency portfolios, and then trace the upper portion of the mean-variance frontiers.

Examining Panel A first, the most notable effect of including investment grade and high yield debt into the investment set is that REIT preferred stock no longer dramatically shifts the mean variance frontier to the left. In fact, inclusion of REIT preferred stock only marginally expands the frontier. While the expansion isn't as dramatic as before, it is still beneficial to highly risk averse investors who are able to attain a higher indifference curve. Low risk aversion investors are unaffected by access to REIT preferred stock, with their portfolios and utility remaining unchanged.

In Panel B, we observe that the inclusion of investment grade and high yield bonds has not changed the effect of REIT common stock on the frontier. Once again, REIT common stock allows the investor to create portfolios with higher returns than they otherwise would have been able to form. This benefits low risk aversion investors, allowing them to attain higher utility levels than before. Notice that REIT common stock providing investors some benefit when the investor has access to both bonds and value portfolios is quite different to the conclusion from the standard case. When we introduce realistic constraints into the portfolio maximization problem, we find that REIT common and preferred stock do play a beneficial role in investor portfolios. It is just that they benefit different investors. Low risk aversion investors benefit from REIT common stock, while high risk aversion investors benefit from REIT preferred stock.[29]

Consistent with what we observed for the case with no bonds, high risk aversion investors still require the majority of their compensation for losing access to REIT preferred stock, and low risk aversion investors require the majority of the compensation for losing access to REIT common stock. This is evident in the compensation ratios reported in Figure 11.

In Figure 12 we report optimal portfolio weights over different levels of risk aversion. The assets are REIT preferred stock (blue), REIT common stock (red), other equity classes (green), investment grade bonds (brown), high yield bonds (orange), and the risk free asset (black.) Panel A reports the case where the investor has access to all asset classes, while Panel B removes REIT common stock, and Panel C removes REIT preferred stock.

In Panel A we observe that for low risk aversion investors, REIT common stock plays a dominant role in their portfolio. As risk aversion increases, REIT preferred stock starts increasing in importance, and for very high levels of risk aversion the optimal portfolio consists of investment grade bonds, the risk free asset, and approximately 10% allocations each to REIT preferred stock and other equity asset classes. Interestingly, in this scenario, the investor makes no use of high yield bonds. Overall, for plausible levels of risk aversion we observe investors holding significant amounts of both REIT common and preferred stock.

When we remove access to REIT common stock in Panel B, the investor allocates slightly more to REIT preferred stock at low levels of risk aversion, and considerably more to the other equity asset classes. Allocations to investment grade bonds and the risk free rate remain similar.

In Panel C, where we remove access to REIT preferred stock, we observe that investors allocate more heavily towards REIT common stock and start to invest in investment grade bonds at lower levels of risk aversion. Notice that if we compare Panel C to Panel A, we observe an interesting relationship between REIT preferred stock, REIT common stock, and investment grade bonds. While access to REIT preferred stock only expanded the frontier modestly in Figure 10 when the investor also had access to investment grade bonds, the allocations to preferred stock in the optimal portfolio suggests that, at least for some investors, preferred stock and bonds are not substitutes. If an investor does not have access to preferred stock they will invest more heavily in investment grade bonds and REIT common stock (especially at moderate levels of risk aversion.) This suggests that REIT preferred stock is some combination of REIT common stock and investment grade bonds. However, if you give the investor access to REIT preferred stock, they will use it instead of the combination of REIT common stock and investment grade bonds. In this sense REIT preferred stock provides the investor with some unique risk return profile that they couldn't otherwise replicate in their portfolio. It also explains in part why REITs issue preferred stock – there is at least some subset of investors to whom they provide benefits that can't be easily replicated using other securities.

[29] We repeated this analysis using a non-REIT preferred index, the BAML Fixed Rate Preferred Index. Interestingly, the REIT preferred index appears to dominate the non-REIT index. In fact, the non-REIT preferred index is spanned by the other available asset classes.

4.2.4 Monte Carlo Evidence

The portfolio allocations presented thus far represent the optimal allocations that investors would hold based on the realized history of returns. It is well known that mean variance optimal portfolios are sensitive to the returns used in the analysis,[30] and thus it is possible that these relationships are confined only to the historic data series we examine. To examine this matter we perform Monte Carlo simulations to create 15,000 different return series. The approach we adopt is the same as in Goetzmann and Ukhov (2006). In order to keep the precision of the covariance matrix the same as in the previous results, we create return series that are also 241 months in length.[31] We do this by drawing one month at a time from the possible 241 months available in the historic return data. To maximize the potential variability in the return data, we draw from the 241 historic months with replacement. For each month drawn, we take returns for all assets in that month. Sampling in this way allows us to generate return series that are considerably different from the historic series used previously, but that also maintain some of the economic covariance structure observed in the historic data.

We report the average long only portfolio allocations for the Monte Carlo experiment in Figure 13. As in the previous figures, the red line is the allocation to REIT common stock, blue is the allocation to REIT preferred stock, green is the allocation to the other equity asset classes, black is the risk free rate, brown is investment grade bonds, and orange is high yield bonds. The 95% confidence interval for each allocation is reported in the dashed lines around the mean allocation.[32] We once again report allocation over different levels of risk aversion. In Panel A we report allocations when the investor has access to all asset classes, in Panel B we report allocations when REIT common stock is removed from the asset set, and in Panel C we report allocations when REIT preferred stock is removed from the asset set.

A few results are noticeable from Figure 13 Panel A. First, the economic relationships observed in the previous results are still present. Investors with low risk aversion have high allocations to equity asset classes and REIT common stock, and as the level of risk aversion increases, investors allocate away from these asset classes towards REIT preferred stock, investment grade debt, high yield debt, and at very high levels of risk aversion to the risk free rate. Second, the mean allocations tend to be less extreme than those observed using the historic data. The allocation to REIT common stock and REIT preferred stock reach a maximum of 41.6% and 31.2% respectively. Although not as extreme as the allocations observed using the historic data, they are still significant over a large range of risk aversion levels. Finally, unlike in the historic data, high yield bonds receive significant allocations over many levels of risk aversion.

In Figure 13 Panel B we can see the mean portfolio allocations when the investor has REIT common stock removed from their asset set. The most obvious change is the increased allocation to equity asset classes especially for low levels of risk aversion. These changes are also reported in Table 3, which reports the changes in allocations between the all asset case and the case where REIT common stock is removed. Less noticeable in the figure, but apparent in the Table 3 is that investors, while increasing their allocation to other equity asset classes, also increase their allocation to REIT preferred stock when REIT common is removed. The change in allocation ranges from 10.44% for very low levels of risk aversion to approximately 2% for extremely high levels of risk aversion. In this sense REIT common stock is not perfectly substituted by other equity asset

[30] See Best and Grauer (1991) and Black and Litterman (1992) for a discussion.
[31] The reason for using longest sample period in our analysis instead of shorter subsamples, is that larger samples tend to have better statistical properties than smaller samples in mean-variance optimizers. See Levy and Kroll (1980) and Kan and Smith (2008) for a discussion.
[32] See Sobol (1972) for a discussion on estimating standard errors in Monte Carlo experiments.

classes, suggesting there is some common real estate characteristic that REIT common and REIT preferred stock possess. Not surprisingly, notice that allocation changes to the debt asset classes (risk free, investment grade, and high yield) are economically quite small.

Figure 13 Panel C reports mean allocations when REIT preferred stock is removed from the investor's asset set, while Table 4 reports changes in allocations. The hybrid nature of preferred stock is quite evident from Table 4. At low levels of risk aversion, an investor increases allocations to REIT common stock and the other equity asset classes, while at higher levels of risk aversion, the investor starts to allocate more heavily towards investment grade and high yield debt.

Notice that while the optimal portfolios reported in Figure 13 may seem extreme, they need to be interpreted in the context of our framework. Because two fund separation breaks down, we are not saying that all investors hold these extreme portfolios, simply that some investor would hold portfolios this extreme if given the chance. Given that we observe quite extreme portfolios being held by individual investors in Barber and Odean (2000), our results do not appear as unreasonable as they would if we were in the classical mean-variance setting and assuming all investors hold the same extreme portfolio.

In Figure 14 we report the mean compensation ratios for the Monte Carlo experiments. We once again see the familiar pattern that the benefits of access to the REIT market are dominated by REIT common stock for a low risk aversion investor and dominated by REIT preferred stock for higher risk aversion investors. This is evident in the downward sloping red line and upward sloping blue line in Figure 14.

The Monte Carlo results thus far only address the issue of how portfolio allocations change, not how valuable access to an asset is to an investor. In Table 5 we report changes in Sharpe Ratios that result from those changed portfolio allocations. The column Preferred refers to the case where REIT preferred stock is added to the asset set, while REIT Common refers to the case where REIT common stock is added, and Preferred & REIT Common refers to the case where both REIT preferred and common stock are added to the asset set.

From Table 5 we observe that having access to REIT preferred stock is quite valuable especially for low risk aversion investors. Having access to REIT preferred stock leads to approximately a 7% improvement in Sharpe Ratio for a low risk aversion investor. Even for an extremely risk averse investor, Sharpe Ratios improve by approximately 3%. Turning to REIT common stock, we observe much smaller changes in Sharpe Ratios. This suggests that access to REIT preferred stock is more important than access to REIT common stock for most investors. Interestingly, the final column of Table 5 demonstrates that having access to both REIT common and Preferred stock leads to greater increases in Sharpe Ratios than simply having access to either REIT preferred stock or REIT common stock separately.

Overall the results from the Monte Carlo simulations are quite supportive of the results obtained using the historic data sample. While the allocations are smaller than observed using the historic data, REIT common and preferred stock receive substantial allocations over plausible levels of risk aversion and having access to these asset classes improves investor Sharpe Ratios.

5 Conclusion

We study diversification benefits of REIT preferred and common stocks using a utility based framework. Employing a utility based framework allows us to examine the diversification benefits of different asset classes, while taking into consideration realistic constraints that investors face. Being unable to short risky assets or borrow at the risk free rate has a material impact on how investors should gauge the benefit of different asset classes. In our utility based framework, investors separate based on risk aversion. That is, two fund separation no longer obtains and investors no longer hold the market portfolio and a long or short position in the risk free asset. This means that standard metrics of diversification benefits, such as Sharpe ratios, are no longer valid measures of utility maximization.

We conduct our analysis using data from the largest time series available, with our data spanning the period November 1992 to November 2012. We allow the investor to choose from a wide variety of asset classes: the World Ex-US stock portfolio, the S&P 500, midcap and small cap indices, midcap and small cap value and growth portfolios, investment grade bonds, high yield bonds, and the risk free asset. We establish three main results. First, while REIT common and preferred stock do provide diversification benefits, they provide them to different investors. REIT common stock allows investors to form higher return portfolios. This is beneficial to low risk aversion investors who seek to form such portfolios. REIT preferred stock is beneficial to high risk aversion investors. REIT preferred stock allows investors to form portfolios with lower risk. This is particularly true if the investor doesn't have access to investment grade bonds. Over realistic levels of risk aversion, we find both REIT common and preferred stock forming substantial portions of an investor's optimal investment portfolio.

Our second main result sheds some light on why firms issue preferred stock. Conventional wisdom is that preferred stock is either a bond substitute or a hybrid security of debt and equity. If this is true, then investors shouldn't change their portfolio allocations when given access preferred stock, if they already have access to these other asset classes. We find that this is not the case. When given access to REIT preferred stock, investors reduce their allocation to investment grade bonds and REIT common stock quite substantially. They are also able to attain higher levels of utility when they have access to preferred stock. In this sense, while the conventional wisdom is true, preferred stock is a hybrid asset, the reality is that the exact risk return characteristics of preferred stock can't easily be replicated by the two other asset classes. Investors would still rather have the ability to invest in preferred stock than be constrained to just common stock and investment grade debt. This is especially true for high risk aversion investors.

Finally, we document empirically that imposing realistic constraints on the investor's portfolio decisions leads to markedly different portfolio allocations that one observes in the standard unconstrained setting. Importantly, these new results help explain economic phenomena, such as why the REIT industry exists even though it appears to be a redundant asset in the classical mean-variance setting, and why REITs issue preferred stock on a large scale.

Monte Carlo experiments indicate that our results are not dependent on the sample period studied. While the portfolio allocation to REIT common and preferred estimated from the historic time series are larger than the average Monte Carlo allocations, the Monte Carlo allocations are both economically and statistically significant.

APPENDIX

To derive the efficient frontier in the standard deviation (x-axis)-return (y-axis) space, trace the frontier by comparing expected utility under different cases. Case 2 (corner solution with all money invested in the riskless asset) is dominated by Case 1 (with positive investment in the risk-free asset and both risky assets).

Case 1. Interior Solution: Positive investment in the risk-free rate and both risky assets: $\lambda_1 = \lambda_2 = \lambda_3 = 0$,

$$w_{rf} = \frac{\mu_2\sigma_1(\sigma_1 - \rho\sigma_2) - r(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2) + a_i W_0(\rho^2 - 1)\sigma_1^2\sigma_2^2 + \mu_1\sigma_2(\sigma_2 - \rho\sigma_1)}{a(\rho^2 - 1)\sigma_1^2\sigma_2^2}$$

$$w_1 = \frac{(\mu_1 - r)\sigma_2 - (\mu_2 - r)\rho\sigma_1}{a_i\, \sigma_1^2\sigma_2(1 - \rho^2)}, \qquad w_2 = \frac{(\mu_2 - r)\sigma_1 - (\mu_1 - r)\rho\sigma_2}{a_i\, \sigma_1^2\sigma_2(1 - \rho^2)}$$

The Lagrange multiplier associated with the budget constraint: $\varphi = 1 + r$.

The investor who hold all his wealth in the tangency portfolio has zero investment in the risk free asset. Risk aversion of this investor can be found by solving $w_{rf} = 0$ and equals,

$$a_{Tangency} = \frac{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho\sigma_1\sigma_2 + (r - \mu_1)\sigma_2^2}{W_0(-1 + \rho^2)\sigma_1^2\sigma_2^2}$$

The allocation to the risk free asset is increasing in risk aversion. To show this, consider the derivative of the allocation to the risk free asset with respect to risk aversion,

$$\frac{\partial w_{rf}}{\partial a_i} = \frac{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho\sigma_1\sigma_2 + (r - \mu_1)\sigma_2^2}{a_i^2(-1 + \rho^2)\sigma_1^2\sigma_2^2}$$

Comparing the above expression for $\frac{\partial w_{rf}}{\partial a_i}$ with the expression for $a_{Tangency} > 0$, it follows that $\frac{\partial w_{rf}}{\partial a_i} > 0$. Therefore, allocation to the risk-free asset is increasing in risk aversion.

Case 2. Corner Solution: All money invested in the riskless asset.

$$w_{rf} = W_0, \lambda_1 = r - \mu_1, \lambda_2 = r - \mu_2, \varphi = \ 1 + r, \lambda_3 = 0, w_1 = 0, w_2 = 0.$$

Under the maintained assumption $r < \mu_1 < \mu_2$, for all positive levels of wealth, $W_0 > 0$ and for all positive levels of risk aversion $a_i > 0$, the expected utility under this solution is lower than the expected utility under case 1 (holding of the risk free asset and both risky assets). This case is therefore dominated by case 1.

Case 3. No investment in the first risky asset, $w_1 = 0$, positive investment in the second risky asset and risk-free asset ($\lambda_2 = 0, \lambda_3 = 0$),

$$w_{rf} = W_0 + \frac{r - \mu_2}{a_i\sigma_2^2}, \lambda_1 = r - \mu_1 + \frac{(\mu_2 - r)\rho\sigma_1}{\sigma_2}, w_2 = \frac{\mu_2 - r}{a_i\sigma_2^2}, \varphi = 1 + r$$

To determine when this case yields higher expected utility than case 1, consider the following inequality,

$$Expected\ Utility\ Under\ Case\ 1 > Expected\ Utility\ Under\ Case\ 3$$

$$-\frac{(r-\mu_2)^2\sigma_1^2 - 2(r-\mu_1)(r-\mu_2)\rho\sigma_1\sigma_2 + ((r-\mu_1)^2 - 2a_i(1+r)W_0(-1+\rho^2)\sigma_1^2)\sigma_2^2}{2a_i(-1+\rho^2)\sigma_1^2\sigma_2^2} > (1+r)W_0 + \frac{(r-\mu_2)^2}{2a_i\sigma_2^2}$$

Under the maintained assumptions $0 < r < \mu_1 < \mu_2,\ a > 0, W_0 > 0,\ 0 < \sigma_1 < \sigma_2,\ -1 < \rho < 1,$ the expected utility under case 3 is always smaller than the expected utility under case 1 (the above inequality holds) and case 3 is dominated by case 1.

In particular, for non-positive correlation, $-1 < \rho \leq 0$ the above inequality always holds under the maintained assumptions $0 < r < \mu_1 < \mu_2, a > 0, W_0 > 0, 0 < \sigma_1 < \sigma_2$. The above inequality also always holds for the case of all correlations, $-1 < \rho < 1$ when the Sharpe ratio of asset two is lower than the Sharpe ratio of asset 1, $\frac{\mu_2 - r}{\sigma_2} < \frac{\mu_1 - r}{\sigma_1}$. Other cases can be considered similarly with the conclusion that case 3 is strictly dominated by case 1 in terms of expected utility for all cases, except the unique case when $\mu_2 = r + (\mu_1 - r)\sigma_2/\rho\sigma_1$ (or, equivalently, $\rho = (\mu_1 - r)\sigma_2/[(\mu_2 - r)\sigma_1]$ when case 3 coincides with case 1.

Case 4. All wealth is invested in the second risky asset, $w_1 = 0, w_2 = W_0, w_{rf} = 0$. The Lagrange multipliers are: $\lambda_1 = -\mu_1 + \mu_2 + a_iW_0(\rho\sigma_1 - \sigma_2)\sigma_2, \lambda_2 = 0, \lambda_3 = \mu_2 - r - a_iW_0\sigma_2^2$, and the Lagrange multiplier for the budget constraint is $\varphi = 1 + \mu_2 - a_iW_0\sigma_2^2$.

This case is preferred to case 7 only for investors with sufficiently low risk aversion,

$$a_i \leq \frac{\mu_2 - \mu_1}{W_0(\sigma_2 - \rho\sigma_1)\sigma_2}.$$

Case 5. No investment in the second risky asset, $w_2 = 0, \lambda_1 = 0, \lambda_3 = 0, \varphi = 1 + r,$

$$w_{rf} = W_0 + \frac{r - \mu_1}{a_i\sigma_1^2}, \qquad \lambda_2 = r - \mu_2 + \frac{(-r + \mu_1)\rho\sigma_2}{\sigma_1}, w_1 = \frac{\mu_1 - r}{a_i\sigma_1^2}$$

To determine when this case yields higher expected utility than case 1, consider the following inequality,

$$Expected\ Utility\ Under\ Case\ 1 > Expected\ Utility\ Under\ Case\ 5$$

Under the maintained assumptions $0 < r < \mu_1 < \mu_2, a > 0, W_0 > 0, 0 < \sigma_1 < \sigma_2, -1 < \rho < 1,$ the above inequality always holds and case 5 is dominated by case 1.

Case 6. Corner solution, all funds invested in the first risky asset, $w_1 = W_0, \lambda_1 = 0, w_2 = 0, w_{rf} = 0, \lambda_2 = \mu_1 - \mu_2 + a_iW_0\sigma_1(\rho\sigma_2 - \sigma_1), \lambda_3 = \mu_1 - r - a_iW_0\sigma_1^2,\ \varphi = 1 + \mu_1 - a_iW_0\sigma_1^2.$

Under the maintained assumption $r < \mu_1 < \mu_2$, when is this solution dominated by case 1 (holding of the risk-free asset and both risky assets)?

(Actually, it is possible to have a risky asset with expected return less than the risk-free rate, $\mu_1 < r$, and can consider this case separately, but the result will still hold).

Case 1 dominates this case when,

$$\mu_{p1} - 0.5\ a_i\ \sigma_{p1}^2 > \mu_{p6} - 0.5\ a_i\ \sigma_{p6}^2$$

$$(1+\mu_1)w_1 + (1+\mu_2)w_2 + (1+r)w_{rf} - \frac{1}{2}a_i(w_1^2\sigma_1^2 + 2w_1w_2\rho\sigma_1\sigma_2 + w_2^2\sigma_2^2) > (1+\mu_1)W_0 - 0.5a_iW_0^2\sigma_1^2$$

Under the maintained assumptions $0 < r < \mu_1 < \mu_2, a > 0, W_0 > 0, 0 < \sigma_1 < \sigma_2, -1 < \rho < 1$, the above inequality always holds and case 6 is dominated by case 1.

Case 7. No investment in the risk free asset, $w_{rf} = 0$, positive amounts of the two risky assets held ($\lambda_1 = \lambda_2 = 0$):

$$w_1 = \frac{\mu_1 - \mu_2 + a_iW_0(\sigma_2^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

$$w_2 = \frac{-\mu_1 + \mu_2 + a_iW_0(\sigma_1^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

$$\lambda_3 = \frac{\mu_1(\sigma_2^2 - \rho\sigma_1\sigma_2) + \mu_2(\sigma_1^2 - \rho\sigma_1\sigma_2) - a_iW_0\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2} - r$$

$$\varphi = \frac{-(2+\mu_1+\mu_2)\rho\sigma_1\sigma_2 + (1+\mu_1)\sigma_2^2 + (1+\mu_2)\sigma_1^2 - a_iW_0\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}$$

Comparing expected utility under case 1 with expected utility under case 7, we conclude that for risk aversion that satisfies,

$$a_i > \frac{(r-\mu_2)\sigma_1^2 + (\mu_1+\mu_2-2r)\rho\sigma_1\sigma_2 + (r-\mu_1)\sigma_2^2}{W_0(-1+\rho^2)\sigma_1^2\sigma_2^2}$$

case 1 is preferred to case 7. That is, investors with sufficiently higher risk aversion hold portfolios described by case 1 and investors with sufficiently low risk aversion hold portfolios described by case 7.

Comparing case 7 (investment in both risky assets) with case 4 (investment in the second risky asset only), shows that investors with sufficiently low risk aversion,

$$a_i \le \frac{\mu_2 - \mu_1}{W_0(\sigma_2 - \rho\sigma_1)\sigma_2},$$

achieve higher expected utility under case 4.

The expressions for portfolio weights w_1 and w_2 are the same as in the case when risk-free asset is not a part of the investment opportunity set.. This case is analogous to the case of no risk-free asset. Therefore, there exists a sufficiently high risk aversion level, $a_i > \frac{2(\mu_2-\mu_1)}{W_0(\sigma_2^2-\sigma_1^2)}$, so that investment in the first (less risk) asset is larger than the investment in the second (more risky) asset, $w_1 > w_2$.

References

Barber, B.M. and T. Odean. 2000. Trading Is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors, *Journal of Finance* 55, 773-806.

Best, M.J., and R.R. Grauer. 1991. On the Sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results, *Review of Financial Studies* 4, 315-342.

Bildersee, J.S. 1973. Some Aspects of the Performance of Non-Convertible Preferred Stocks. *Journal of Finance.* 28, 1187-1201.

Black, F. 1972. Capital Market Equilibrium with Restricted Borrowing. *Journal of Business*, 45, 444-455.

Black, F., and R. Litterman. 1992. Global Portfolio Optimization, *Financial Analysts Journal*, 48, 28-43.

Blackburn D., Goetzmann, W.N., and A.D. Ukhov. 2009. Risk Aversion and Clientele Effects, *NBER Working Paper* 15333.

Bliss, R.R., and Panigirtzoglou, N. 2004. Option-implied Risk Aversion Estimates. *Journal of Finance* 59, 407-446.

Boudry. W.I. 2011. An Examination of REIT Dividend Payout Policy. *Real Estate Economics* 39(4) 601-634.

Boudry W.I., Coulson, N.E, Kallberg J.G., and C.H. Liu. 2013. On Indexing Commercial Real Estate Properties and Portfolios, *Journal of Real Estate Finance and Economics* 47, 617-639.

Boudry W.I., Kallberg J.G., and C.H. Liu. 2010. An Analysis of REIT Security Issuance Decisions. *Real Estate Economics 38*, 91-120.

Bris, A., Goetzmann, W.N., and N. Zhu. 2007. Efficiency and the Bear: Short Sales and Markets Around the World. *Journal of Finance.* 62, 1029-1079.

Chiang, K.C.H, and M.L. Lee. 2007. Spanning tests on public and private real estate, *Journal of Real Estate Portfolio Management* 13, 7-15.

Corgel, J. B. and J.A. deRoos. 1999. Recovery of Real Estate Returns for Portfolio Allocation, *Journal of Real Estate Finance and Economics*. 18(3), 279-296.

Dybvig, P. H. 1984. Short Sales Restrictions and Kinks of the Mean Variance Frontier, *Journal of Finance* 39, 239-244.

Edelstein, R.H. and K. Magin. 2013. The equity risk premium for securitized real estate: the case for U.S. real estate investment trusts. *Journal of Real Estate Research* 35, 393-406

Emanuel, D. 1983. A Theoretical Model for Valuing Preferred Stock. *Journal of Finance*. 38, 1133-1155.

Ferreira, E. J., Spivey, M.F., and C.E. Edwards. 1992. Pricing New-Issue and Seasoned Preferred Stocks: A Comparison of Valuation Models. *Financial Management*. 21, 52-62.

Folger, H.R. 1984. Twenty Percent in Real Estate: Can Theory Justify It? *Journal of Portfolio Management* 10(2), 6-13.

Frost, P.A., and J.E. Savarino. 1988. For better performance: Constrain portfolio weights, *Journal of Portfolio Management* 15, 29-34

Geczy, C.C., Musto, D.K., and A.V. Reed. 2002. Stocks are special too: an analysis of the equity lending market, *Journal of Financial Economics.* 66, 241-269.

Goetzmann, W. and A. Kumar. 2008. Equity Portfolio Diversification, *Review of Finance* 12, 433-463.

Goetzmann, W.N, and A. Ukhov. 2006. British Investment Overseas 1870--1913: A Modern Portfolio Theory Approach, *Review of Finance* 10, 261-300.

Graham, B., and D. Dodd. 1934. *Security Analysis.* McGraw-Hill.

Gyourko, J., and E.F. Nelling. 1996. Systematic Risk and Diversification in the Equity REIT Market. *Journal of Real Estate Economics. 24*(4), 493-515.

Heinkel R., and J. Zechner. 1990. The Role of Debt and Preferred Stock as a Solution to Adverse Investment Incentives. *Journal of Financial and Quantitative Analysis 25*, 1-24.

Ibbotson, R.G., and L.B. Siegel. 1984. Real Estate Returns: A Comparison with Other Investments. *Journal of Real Estate Economics* (AREUEA Journal) *12*:3, 219-242

Jagannathan, R., and T. Ma. 2003. Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps. *Journal of Finance* 63, 1651-1683.

Kallberg, J.G., Liu, C.H., and W. Greig. 1996. The Role of Real Estate in the Portfolio Allocation Process. *Journal of Real Estate Economics. 24*:3, 359-377.

Kallberg, J.G., Liu, C.H., and C. Trzcinka. 2000. The Value Added from Investment Managers: An Examination of Funds of REITs. *Journal of Financial and Quantitative Analysis, 35*(3), 387-408.

Kallberg, J.G., Liu, C.H., and S. Villupuram. 2013. Preferred Stock: Some Insights into Capital Structure. *Journal of Corporate Finance* 21, 77-86.

Kan, R., and D.R. Smith. 2008. The Distribution of the Sample Minimum-Variance Frontier, *Management Science*, 54, 1364-1380.

Kroll, Y., Levy, H., and H.M. Markowitz. 1984. Mean-Variance Versus Direct Utility Maximization. *Journal of Finance*, 39, 47-61.

Lee, H., and D.T. Johnson. 2009. The Operating Performance of Preferred Stock Issuers. *Applied Financial Economics*, 19(5), 397.

Levy, H. and Y. Kroll. 1980. Sample vs. Population Mean-Variance Efficient Portfolios, *Management Science*, 26, 1108-1116.

Lin, C.Y., and K. Yung. 2006. Equity Capital Flows and Demand for REITs. *Journal of Real Estate Finance and Economics 33*:3, 275-291.

Markowitz, H. (1952) Portfolio Selection, *Journal of Finance* 7, 77-91.

Merton, R. C. (1972) An Analytical Derivation of the Efficient Portfolio Frontier, *Journal of Financial and Quantitative Analysis* **7**, 1851-72.

Miles, M.E., and T.E. McCue. 1984. Commercial Real Estate Returns. *Journal of Real Estate Economics*. (AREUEA Journal) *12*:3, 355-377

Ott S. H., Riddiough T.J., and H.C. Yi. 2005. Finance, Investment and Investment Performance: Evidence from the REIT Sector. *Real Estate Economics*. *33*(1), 203-235.

Ross, S.A. 1977. The Capital Asset Pricing Model (CAPM), Short-Sales Restrictions and Related Issues, *Journal of Finance*, 32, 177-183.

Ross. S. and R. Zisler. 1991. Risk and Return in Real Estate. *Journal of Real Estate Finance and Economics* 4(2), 175-190.

Sa-Aadu, J., Shilling, J., and A. Tiwari. 2010. On the Portfolio Properties of Real Estate in Good Times and Bad Times. *Real Estate Economics, 38*(3), 529-565.

Sharpe, W.F. 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance*, 19(3), 425-442.

Sobol, I.M. *Monte Carlo Method*. Nauka, Moscow.

Sorensen E.H., and C.A. Hawkins. 1981. On the Pricing of Preferred Stock. *Journal of Financial and Quantitative Analysis*. 515-528

Ukhov, A.D. (2006) Expanding the frontier one asset at a time, *Finance Research Letters* 3, 194-206.

Yaman, D. 2011. Long-Run Operating Performance of Preferred Stock Issuers. *The International Journal of Business and Finance Research, 5*(2), 61-73.

Figure 1: Constrained Efficient Frontier

The figure shows the constrained efficient frontier. Portfolio weights for all risky assets and the risk free asset are constrained to be non-negative. Segment 1 is the straight line that connects the risk free rate on the left and the tangency portfolio on the right. Segment 2 is the efficient frontier curve constructed with risky assets only. Segment 3 is the portfolio with the highest attainable risk (return).

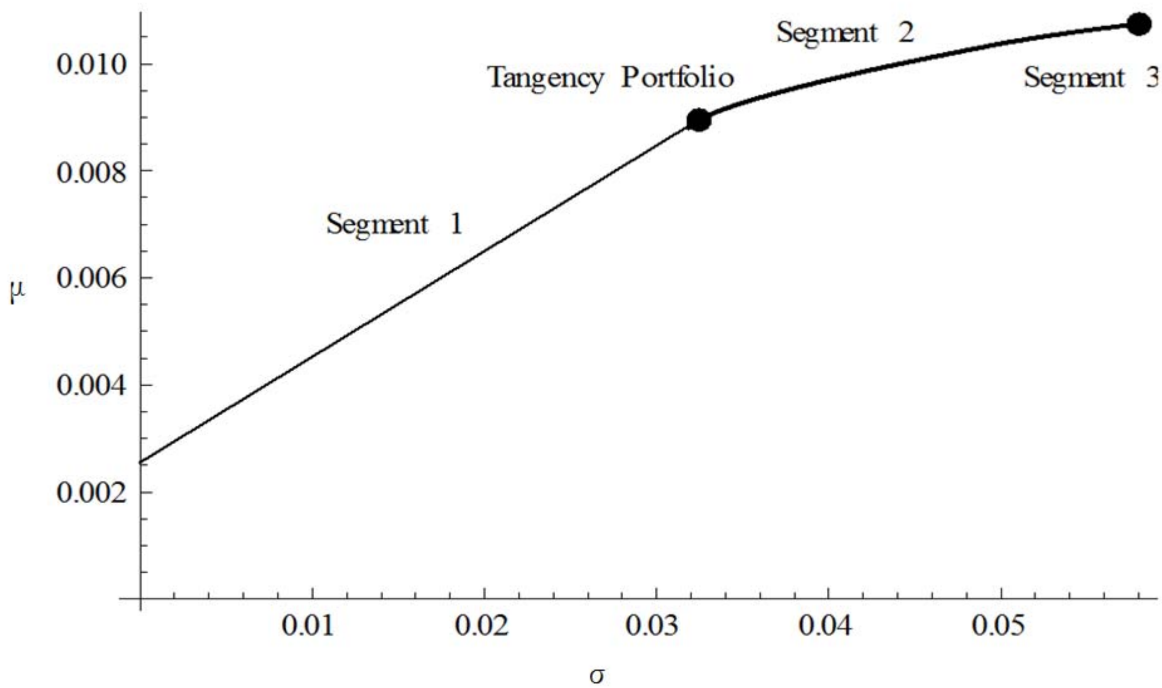
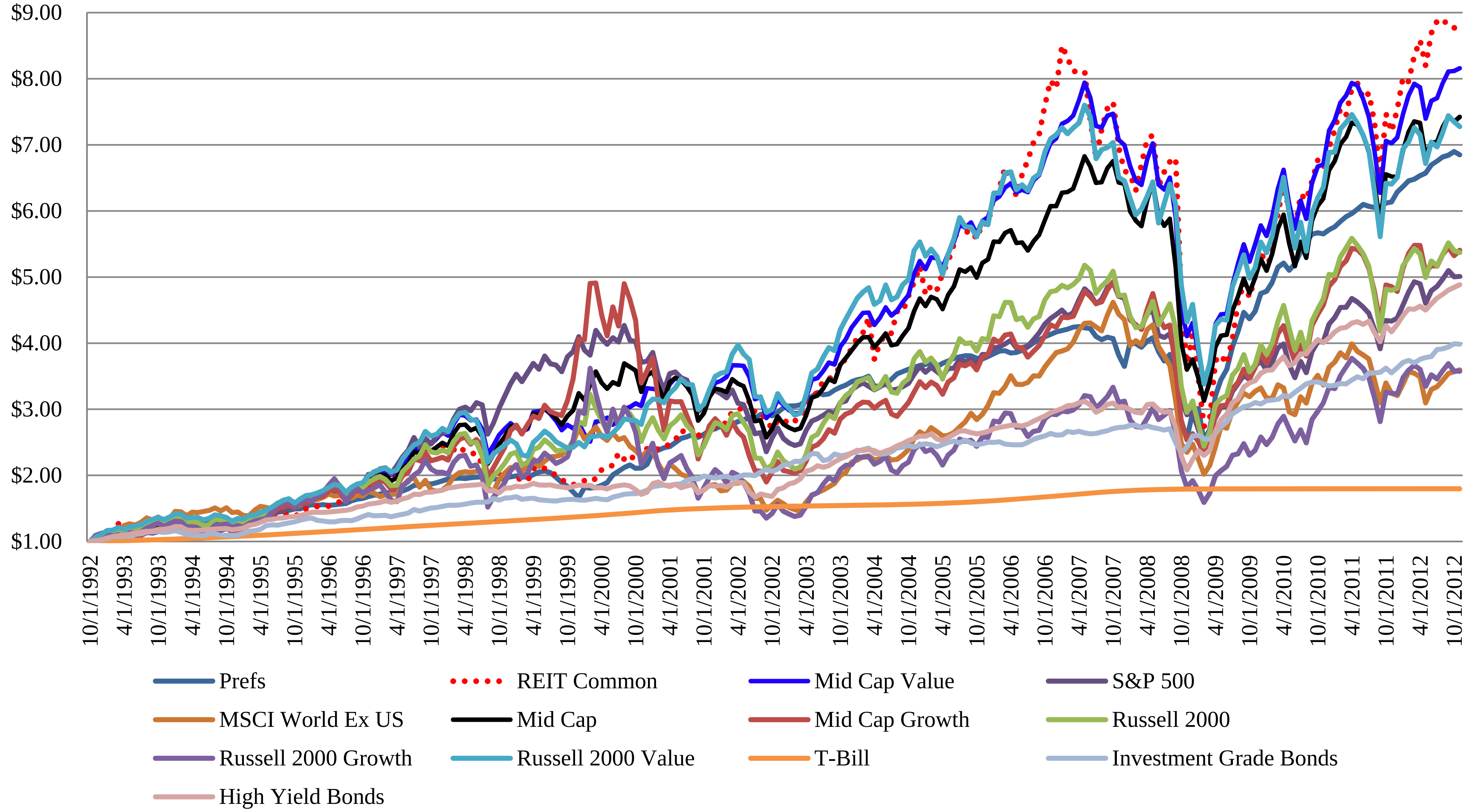


Figure 2.
Performance of Asset Classes
Value of $1 invested on October 1, 1992



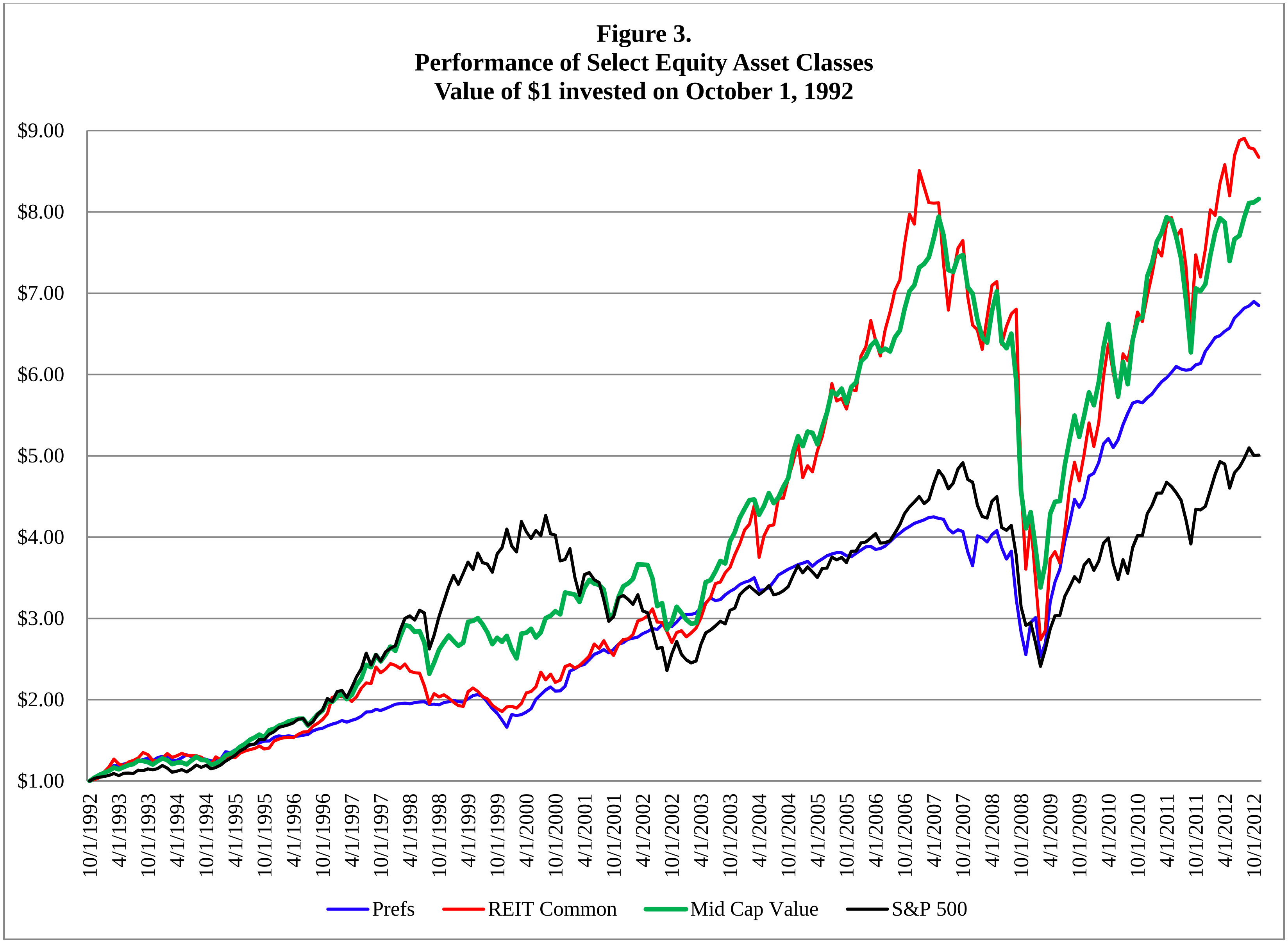


Figure 3.
Performance of Select Equity Asset Classes
Value of $1 invested on October 1, 1992
$9.00
$8.00
$7.00
$6.00
$5.00
$4.00
$3.00
$2.00
$1.00
10/1/1992
4/1/1993
10/1/1993
4/1/1994
10/1/1994
4/1/1995
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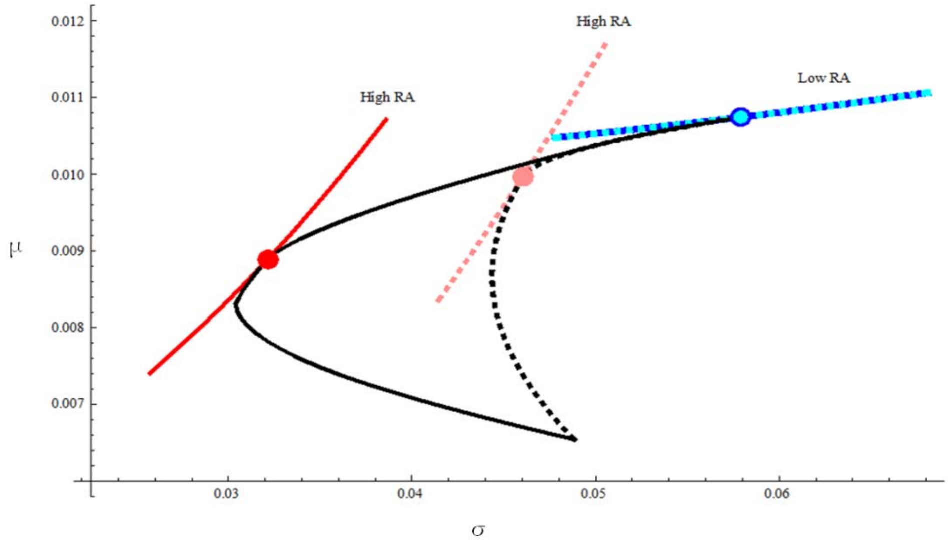
4/1/2010
10/1/2010
4/1/2011
10/1/2011
4/1/2012
10/1/2012
Prefs
REIT Common
Mid Cap Value
S&P 500

Figure 4: Equity Only and No Risk Free Asset
Figure displays mean-variance frontiers (black) and indifference curves for high (red) and low (blue) risk aversion investors. In Panel A, the dashed line represents the case where the investor has access to all equity asset classes but excludes REIT preferred stock, while the solid line presents the case where REIT preferred stock is also included. In Panel B, the dashed line represents the case where the investor has access to all equity asset classes but excludes REIT common stock, while the solid line presents the case where REIT common stock is also included. Data are monthly and span the period November 1992 to November 2012.

Panel A



Panel B

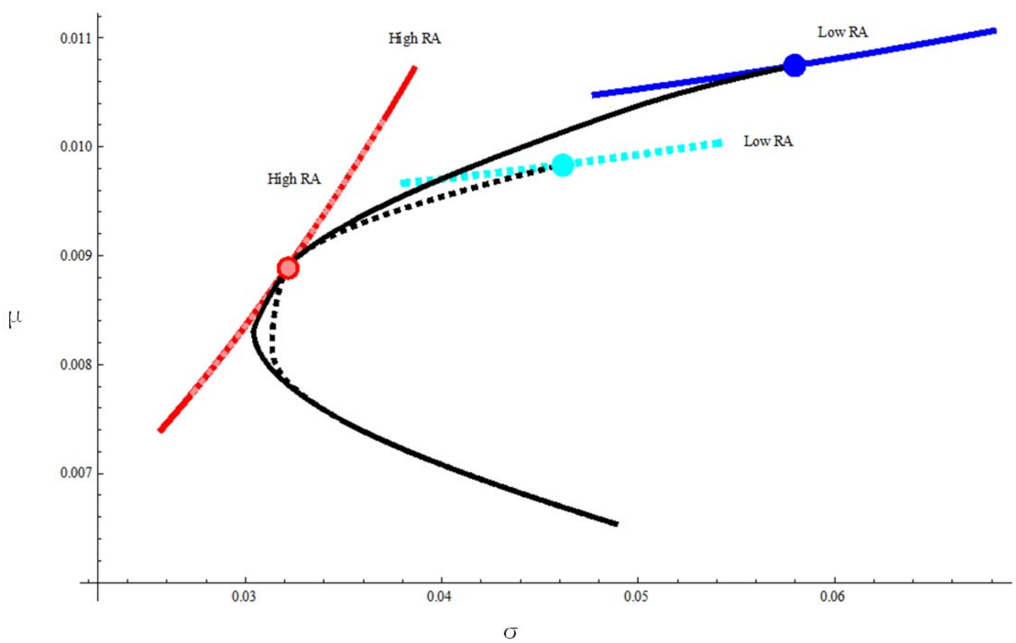


Figure 5: Compensation Ratios (Equity Only and No Risk Free Asset)

The figure reports compensation ratios for different levels of risk aversion. Compensation ratios are calculated as the dollar compensation required to compensate the investor for losing access to REIT common stock (red) and REIT preferred stock (blue) divided by the compensation required to compensate the investor for losing access to both REIT common and preferred stock. The investment opportunity set is the same as in Figure 4.

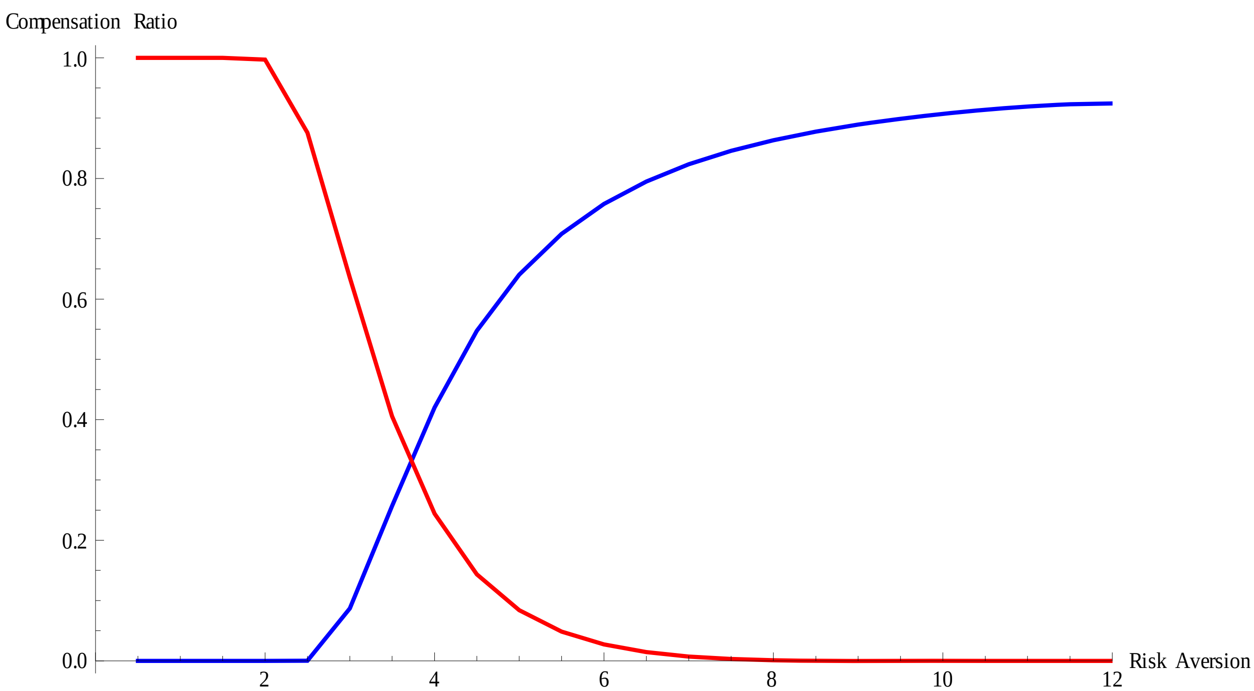
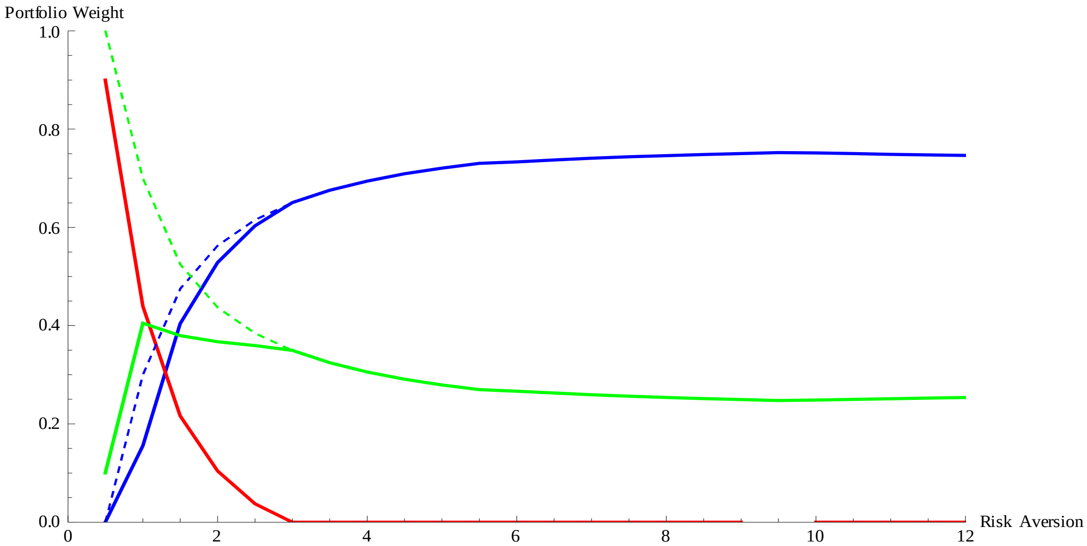


Figure 6: Optimal Portfolio Weights (Equity Only and No Risk Free Asset)
The figure displays optimal portfolio weights for different levels of risk aversion. Red is the weight of REIT common stock, blue is the weight of REIT preferred stock, and green is the weight of all other equity asset classes. Solid lines represent the weights when the investor has access to all equity classes plus REIT common and preferred stock. In Panel A, the dashed line represents weights when the investor loses access to REIT common stock, and in Panel B the dashed lines represent weights when the investor loses access to REIT preferred stock.

Panel A



Panel B

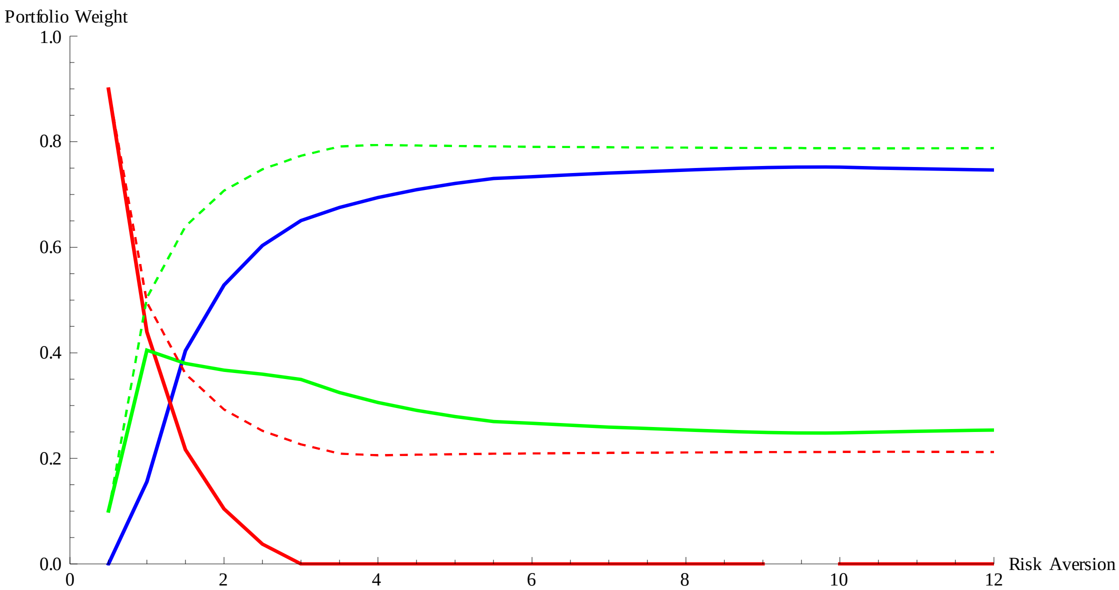
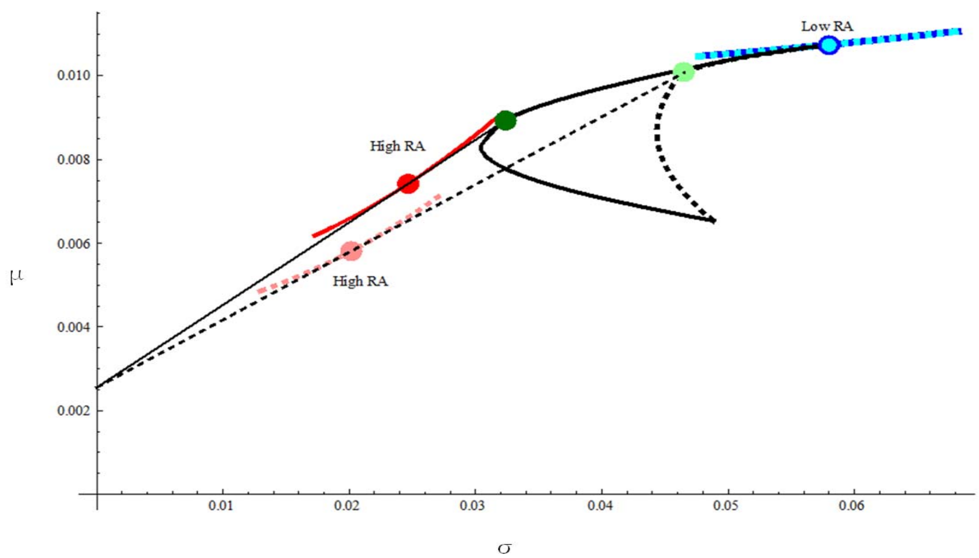


Figure 7: Long Only Risk Free Asset and No Bonds

Figure displays mean-variance frontiers (black) and indifference curves for high (red) and low (blue) risk aversion investors. In Panel A, the dashed line represents the case where the investor has access to all equity asset classes but excludes REIT preferred stock, while the solid line presents the case where REIT preferred stock is also included. In Panel B, the dashed line represents the case where the investor has access to all equity asset classes but excludes REIT common stock, while the solid line presents the case where REIT common stock is also included. The investor has access to the risk free asset and tangency portfolios are given by green circles. Data are monthly and span the period November 1992 to November 2012.

Panel A



Panel B

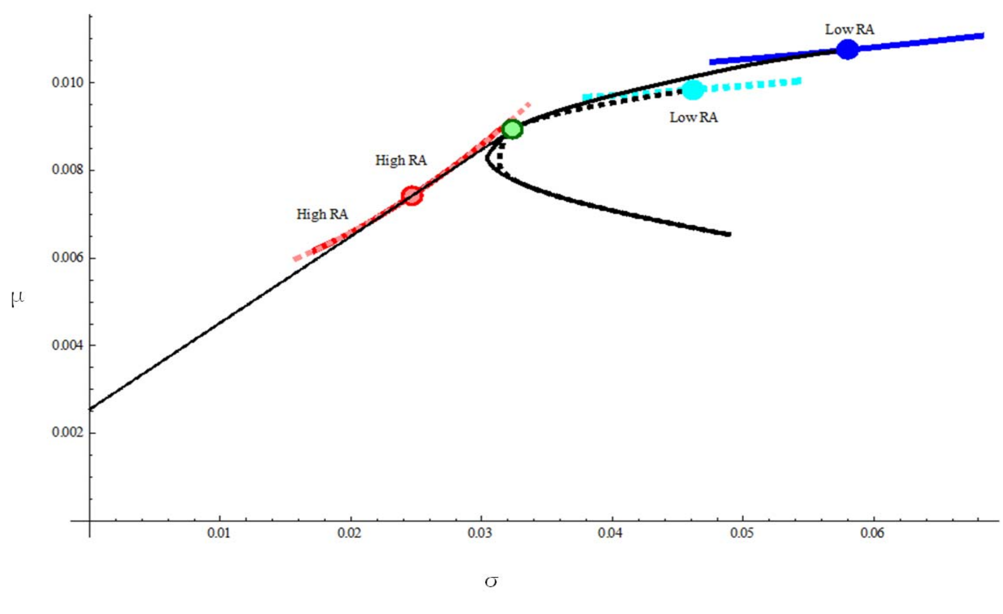


Figure 8: Compensation Ratios (Long Only Risk Free Asset and No Bonds)

The figure reports compensation ratios for different levels of risk aversion. Compensation ratios are calculated as the dollar compensation required to compensate the investor for losing access to REIT common stock (red) and REIT preferred stock (blue) divided by the compensation required to compensate the investor for losing access to both REIT common and preferred stock. The investment opportunity set is the same as in Figure 7.

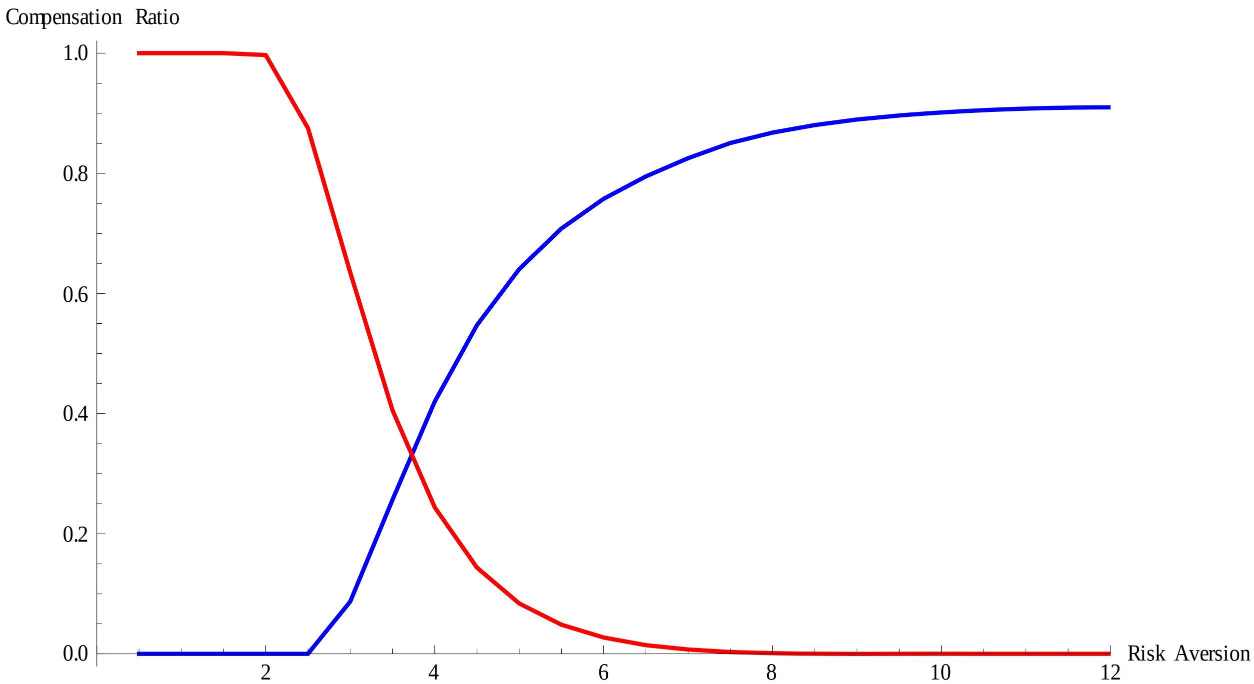
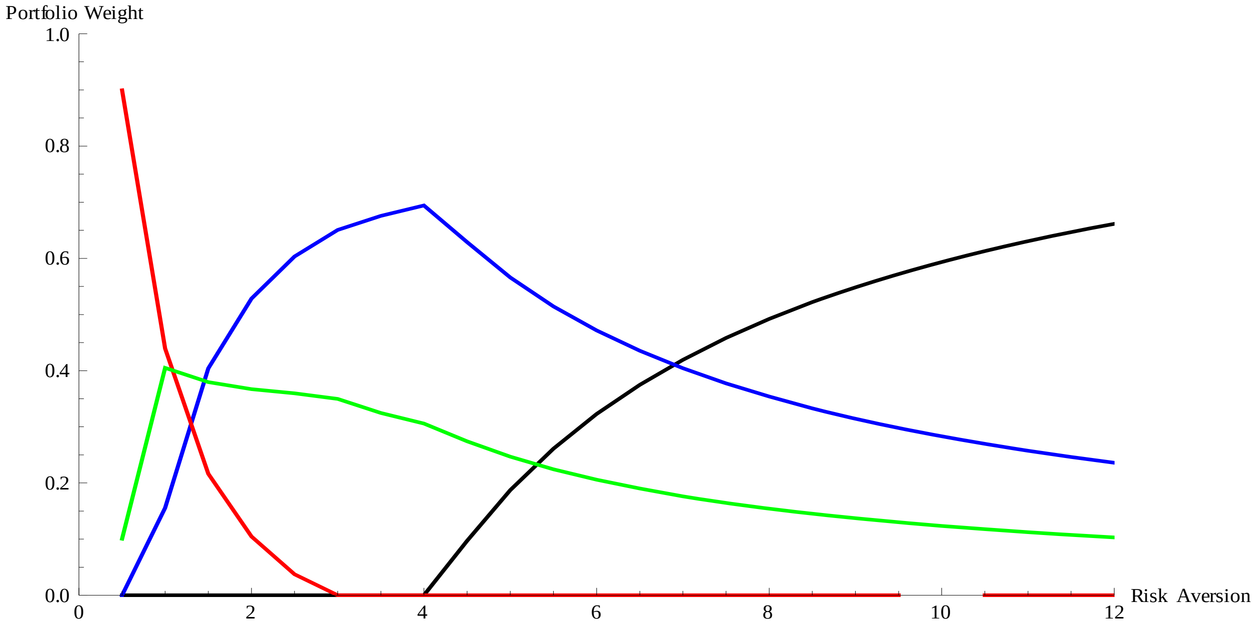


Figure 9: Optimal Portfolio Weights (Long Only Risk Free Asset and No Bonds)
The figure displays optimal portfolio weights for different levels of risk aversion. Red is the weight of REIT common stock, blue is the weight of REIT preferred stock, green is the weight to all other equity asset classes, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B shows weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock.

Panel A



Panel B

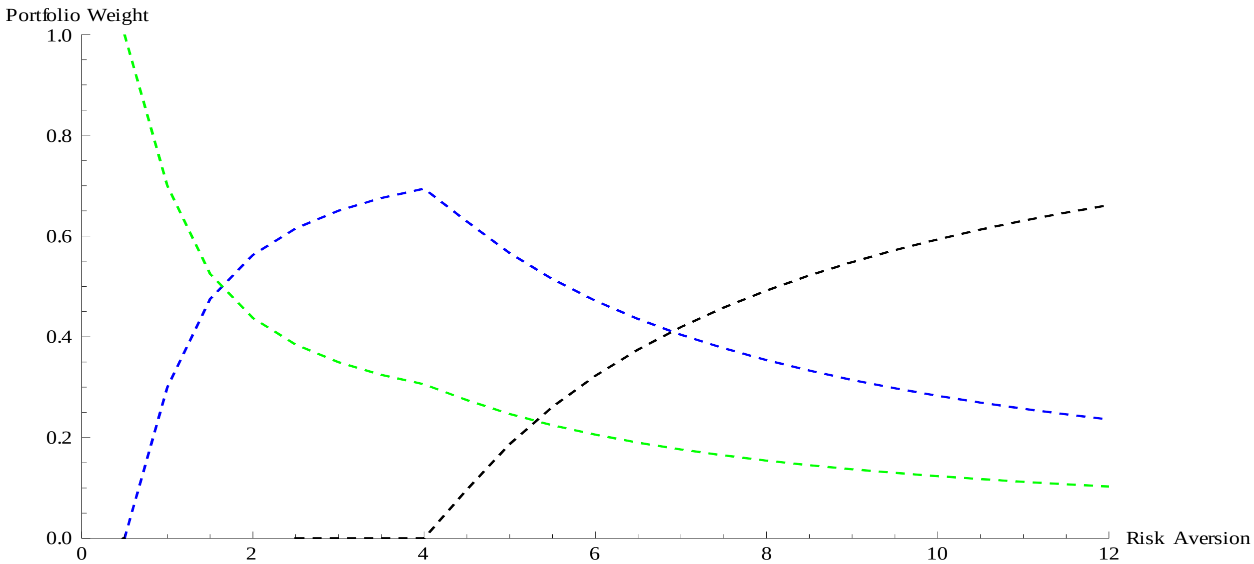


Figure 9: Optimal Portfolio Weights (Long Only Risk Free Asset and No Bonds) continued
The figure displays optimal portfolio weights for different levels of risk aversion. Red is the weight of REIT common stock, blue is the weight of REIT preferred stock, green is the weight to all other equity asset classes, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B shows weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock.

Panel C

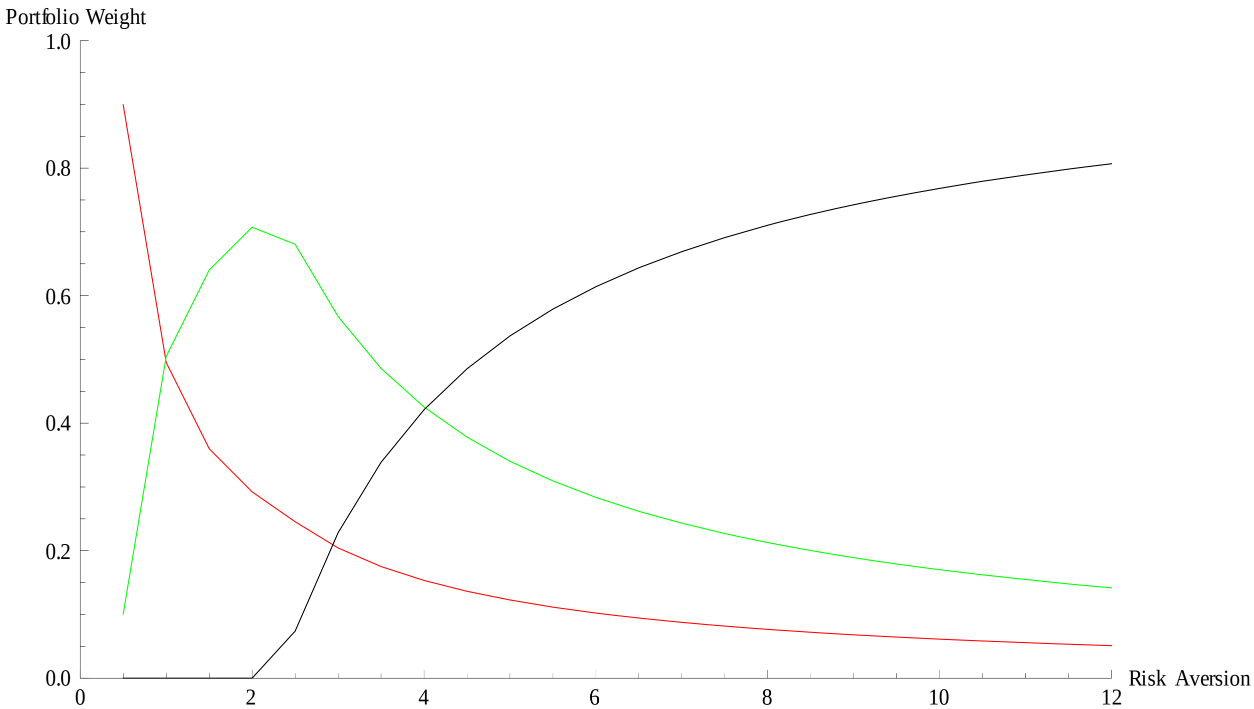
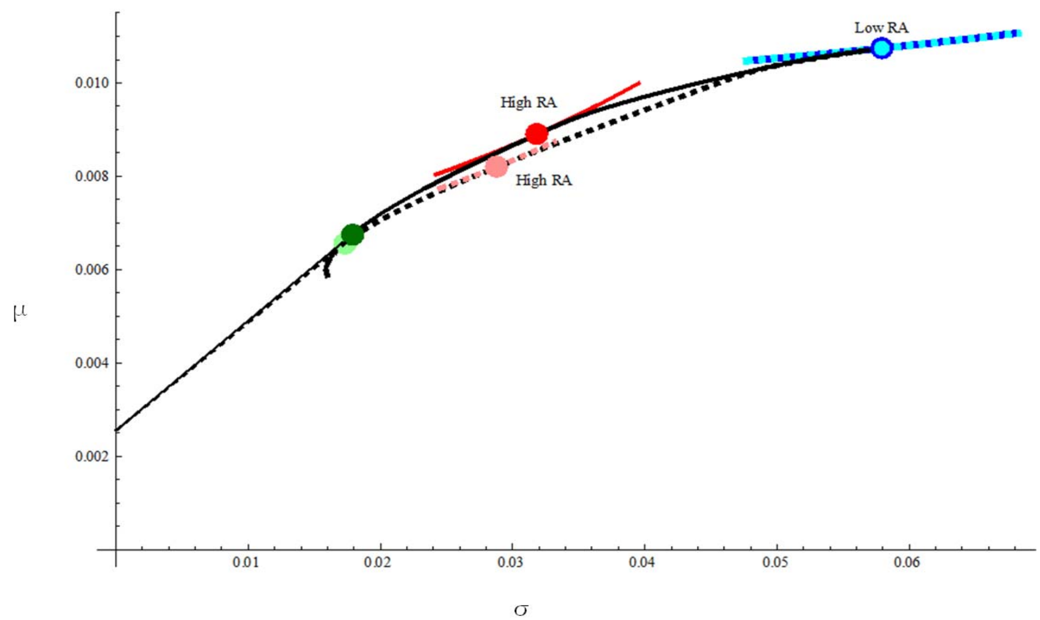


Figure 10: Long Only Risk Free Asset Plus Bonds

Figure displays mean-variance frontiers (black) and indifference curves for high (red) and low (blue) risk aversion investors. In Panel A, the dashed line represents the case where the investor has access to all equity asset classes and investment grade and high yield bonds, but excludes REIT preferred stock, while the solid line presents the case where REIT preferred stock is also included. In Panel B, the dashed line represents the case where the investor has access to all equity asset classes and investment grade and high yield bonds, but excludes REIT common stock, while the solid line presents the case where REIT common stock is also included. The investor has access to the risk free asset and tangency portfolios are given by green circles. Data are monthly and span the period November 1992 to November 2012.

Panel A



Panel B

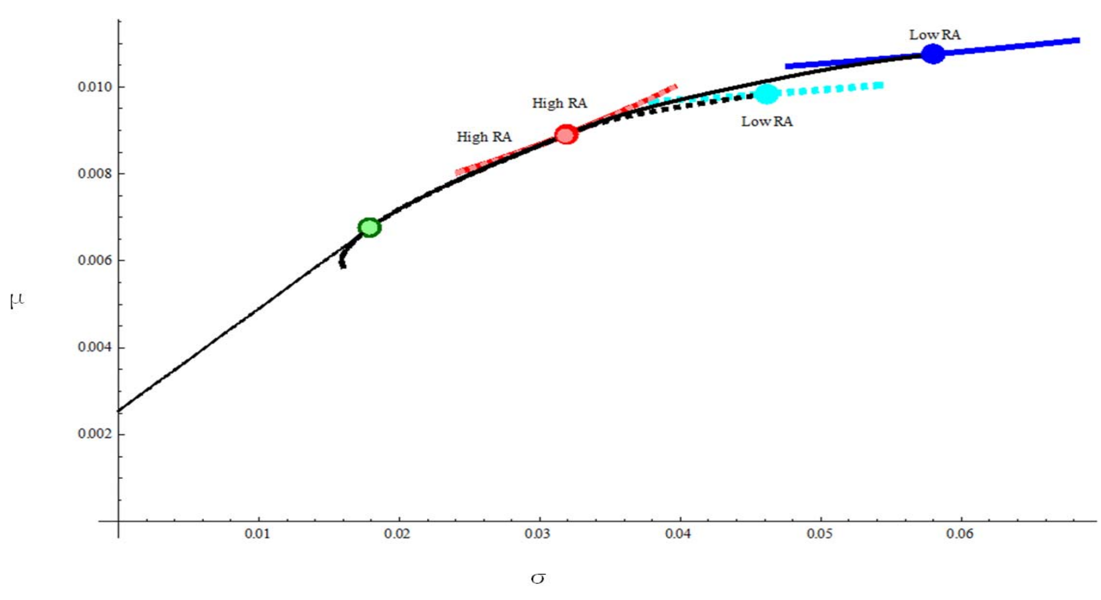


Figure 11: Compensation Ratios (Long Only Risk Free Asset Plus Bonds)

The figure displays compensation ratios for different levels of risk aversion. Compensation ratios are calculated as the dollar compensation required to compensate the investor for losing access to REIT common stock (red) and REIT preferred stock (blue) divided by compensation required to compensate the investor for losing access to both REIT common and preferred stock. The investment opportunity set is the same as in Figure 10.

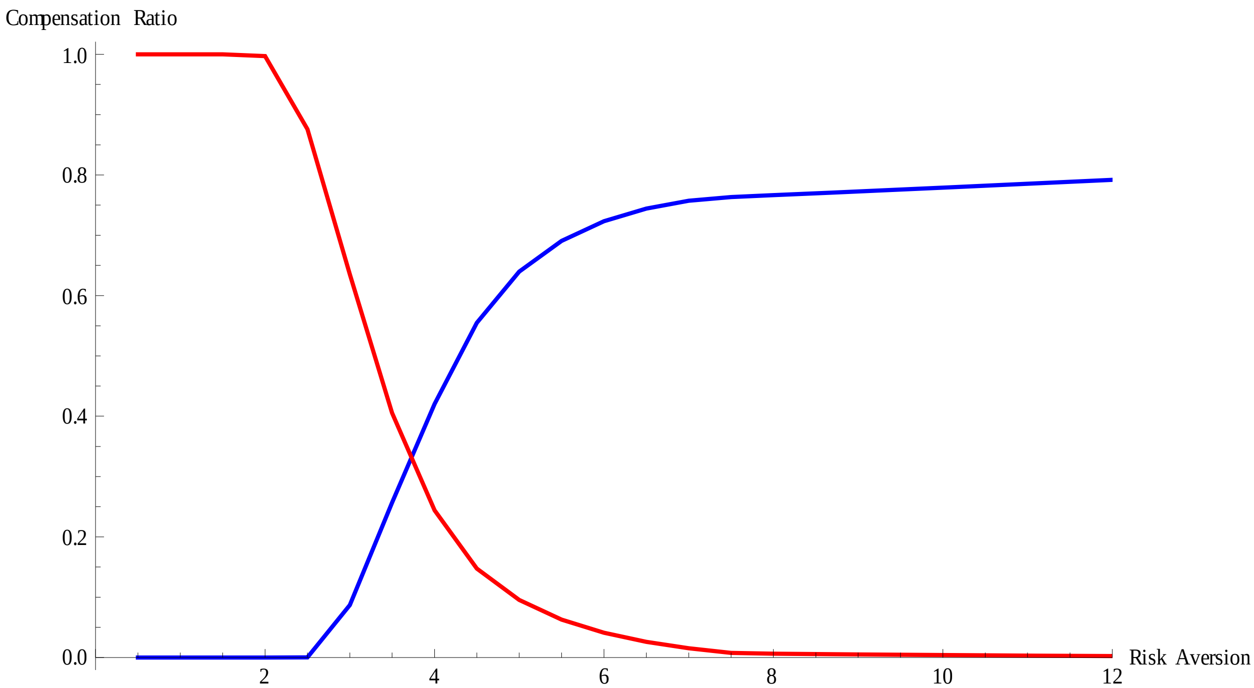
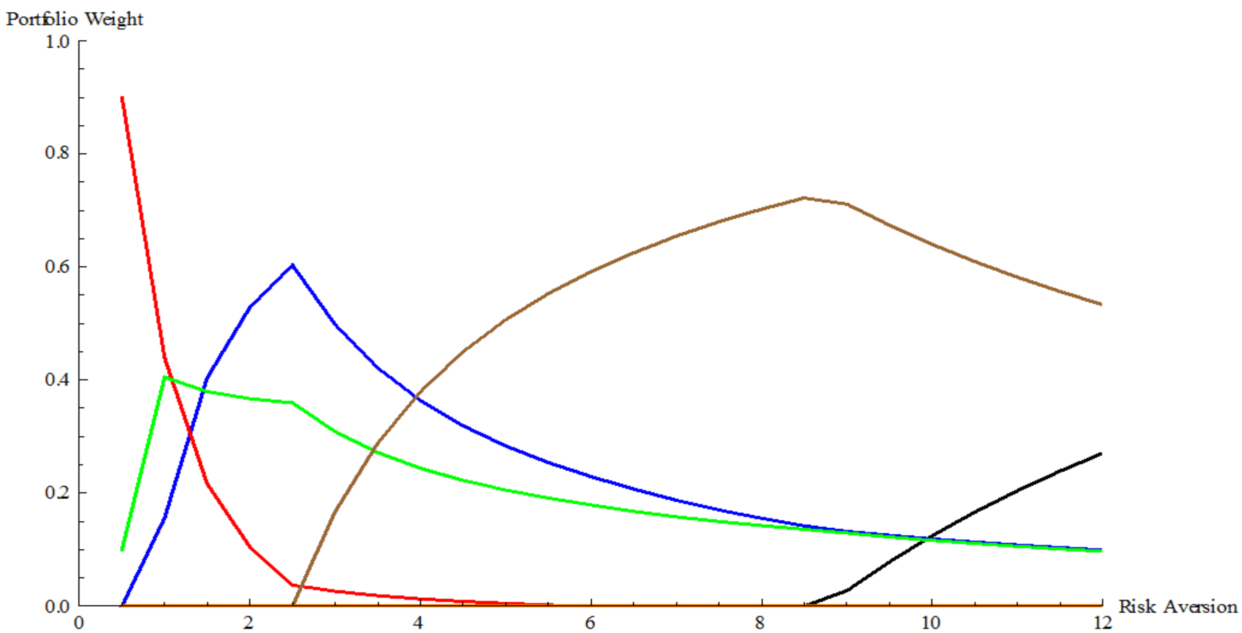


Figure 12: Optimal Portfolio Weights (Long Only Risk Free Asset Plus Bonds)

Figure reports optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, green is the weight to all other equity asset classes, brown is the weight in investment grade bonds, orange is the weight in high yield bonds, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B, reports weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock.

Panel A



Panel B

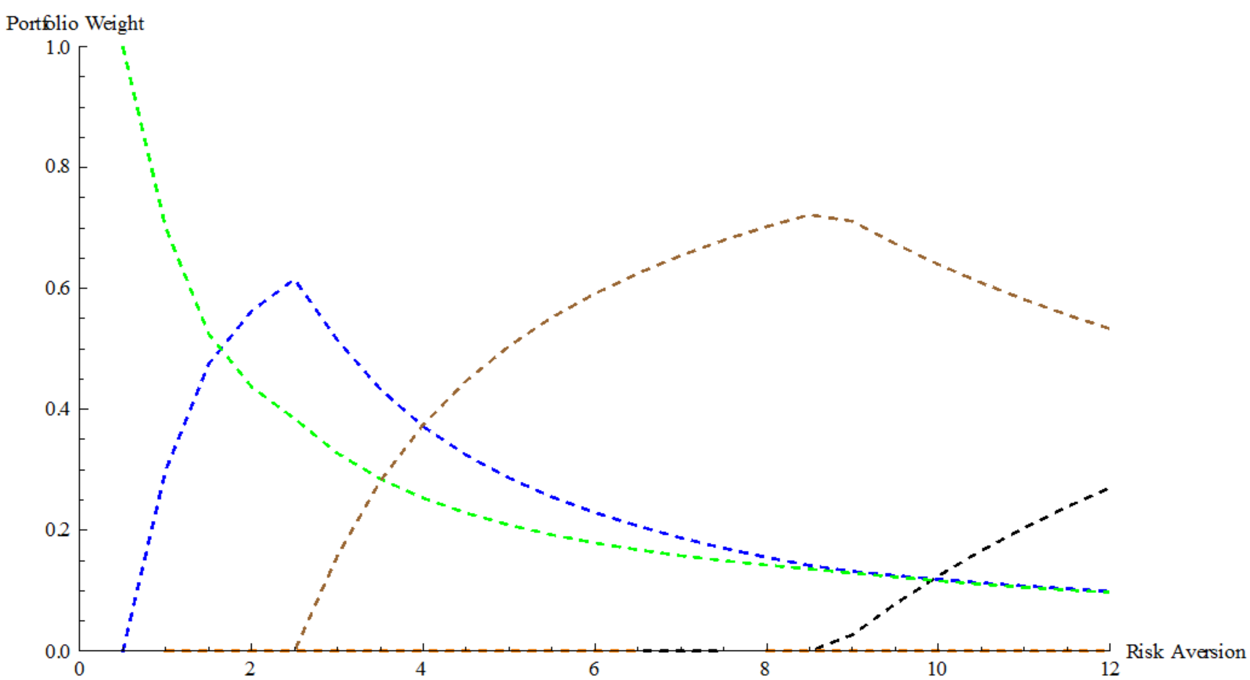


Figure 12: Optimal Portfolio Weights (Long Only Risk Free Asset Plus Bonds) continued

Figure reports optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, green is the weight to all other equity asset classes, brown is the weight in investment grade bonds, orange is the weight in high yield bonds, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B, reports weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock.

Panel C

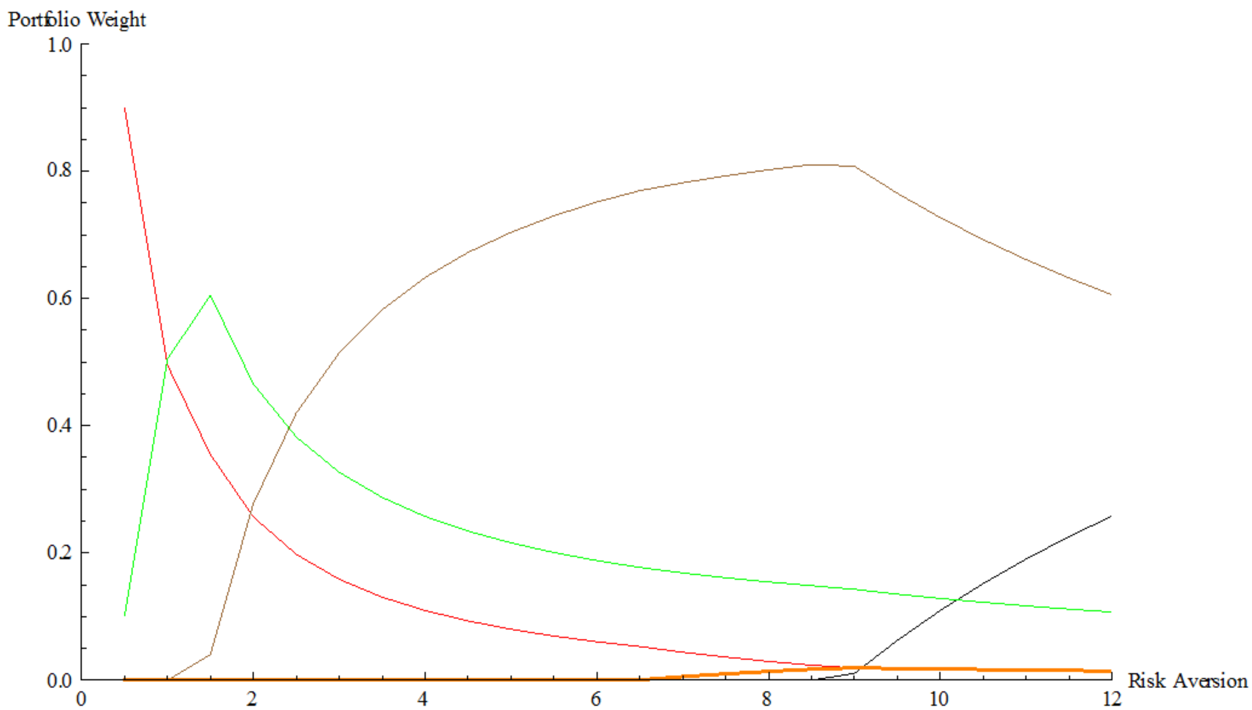
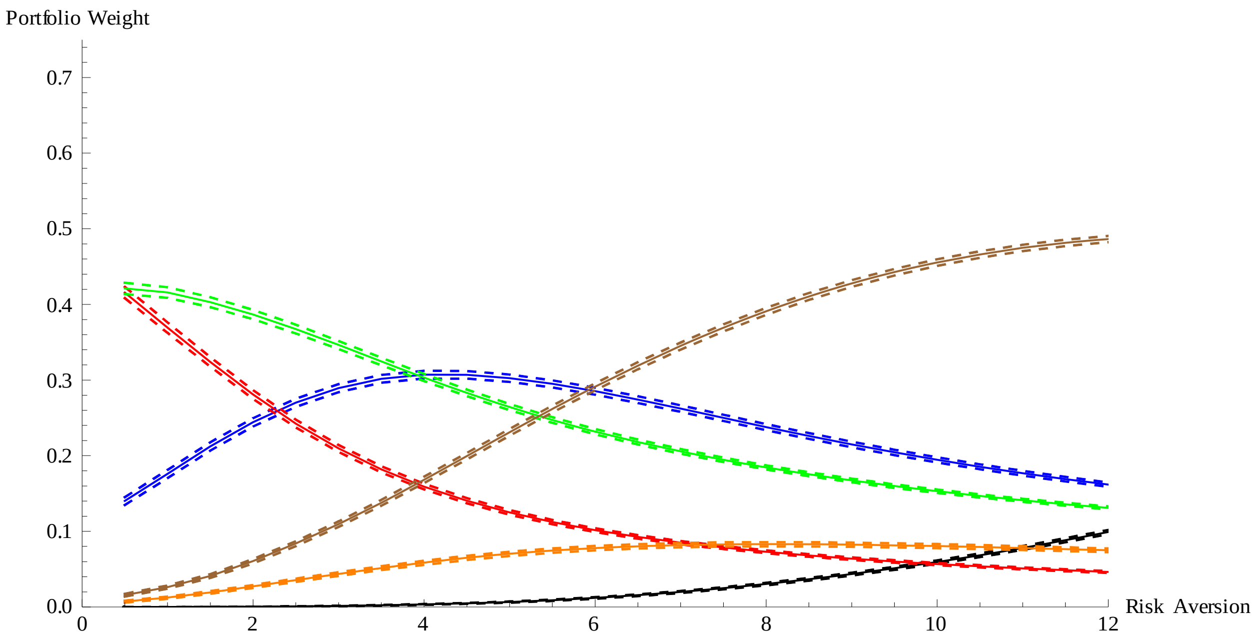


Figure 13: Optimal Portfolio Weights from Monte Carlo Experiment

Figure reports long only optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, green is the weight to all other equity asset classes, brown is the weight to investment grade bonds, orange is the weight to high yield bonds, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B, reports weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock. Weights are mean portfolio allocations from 15,000 Monte Carlo simulations. 95% confidence intervals are reported in dashed lines.

Panel A



Panel B

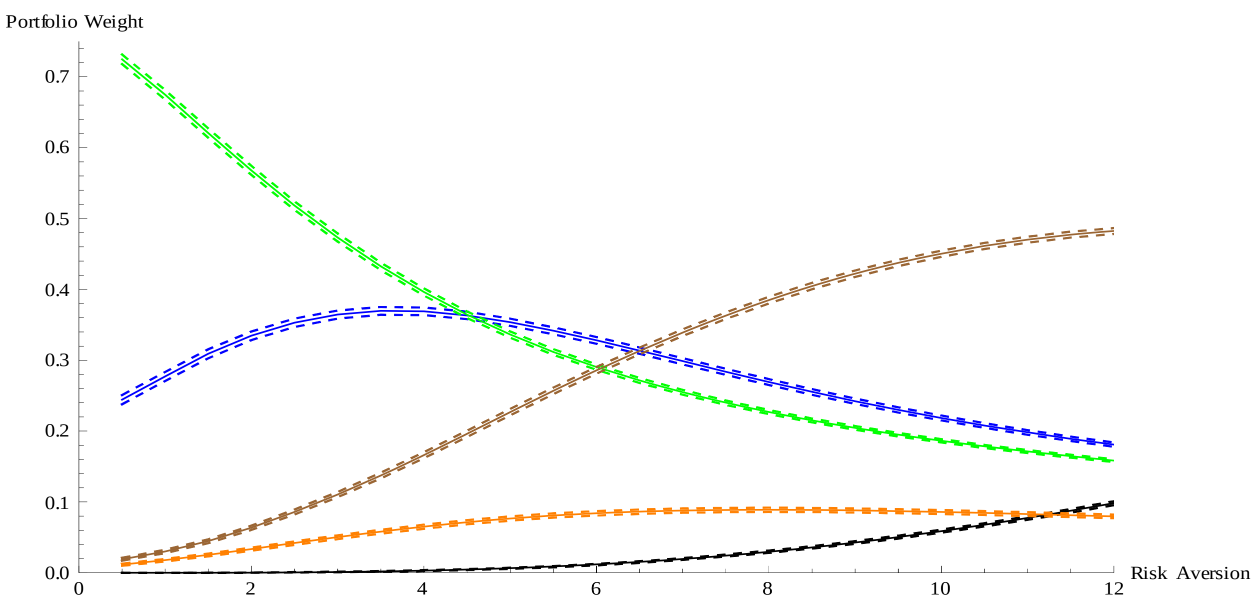


Figure 13: Optimal Portfolio Weights from Monte Carlo Experiment continued

Figure reports long only optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, green is the weight to all other equity asset classes, brown is the weight to investment grade bonds, orange is the weight to high yield bonds, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B, reports weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock. Weights are mean portfolio allocations from 15,000 Monte Carlo simulations. 95% confidence intervals are reported in dashed lines.

Panel C

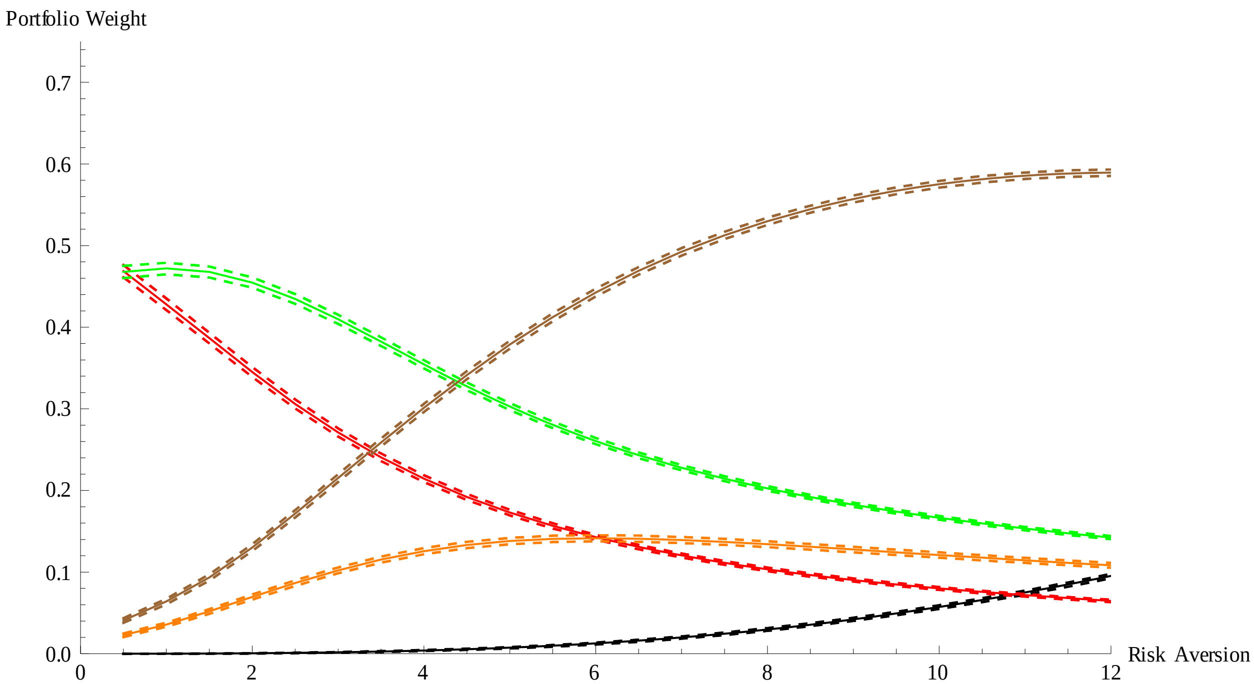


Figure 14: Mean Compensation Ratios from Monte Carlo Experiment

The figure displays compensation ratios for different levels of risk aversion. Compensation ratios are mean values from 15,000 Monte Carlo simulations. In each simulation compensation ratios are calculated as the dollar compensation required to compensate the investor for losing access to REIT common stock (red) and REIT preferred stock (blue) divided by compensation required to compensate the investor for losing access to both REIT common and preferred stock. The investment opportunity set includes REIT common, REIT preferred, several equity asset classes, investment grade bonds, high yield bonds, and risk free asset (the investment opportunity set is the same as in Figure 13).

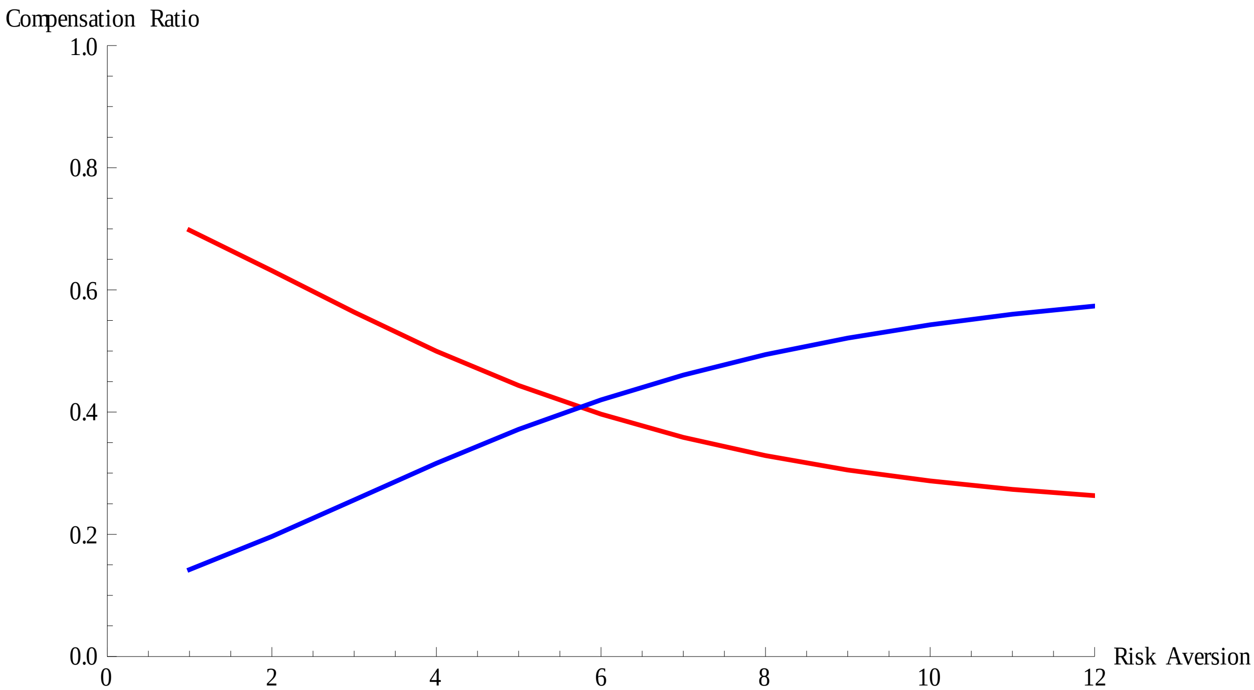


Table 1: Returns and Correlations

Table reports descriptive statistics of monthly returns on 13 indices between November 1992 and November 2012. Panel A reports monthly and annualized means and standard deviations, while Panel B reports correlations. Pref is the MSCI REIT Preferred index, IG Corp is the Barclays Investment Grade Corporate Bond index, HY Corp is the Barclays High Yield Corporate Bond index, World Ex-US is the MSCI World Ex-US index, REIT is the SNL Equity REIT index, MidCap is the Russell MidCap index, MidCap Growth is the Russell MidCap Growth index, MidCap Value is the Russell MidCap Value index, Russ2000 is the Russell 2000 index, Rus2000 Growth is the Russell 2000 Growth index, Rus2000 Value is the Russell 2000 value index, and SP500 us the S&P 500 index.

Panel A

	Pref	IG Corp	HY Corp	World Ex-US	REIT	MidCap	MidCap Growth	MidCap Value	Rus2000	Rus2000 Growth	Rus2000 Value	SP500
Monthly												
Mean	0.009	0.006	0.007	0.007	0.011	0.010	0.009	0.010	0.009	0.008	0.010	0.008
Std	0.033	0.016	0.026	0.049	0.058	0.049	0.062	0.046	0.057	0.067	0.050	0.044
Annualized												
Mean	0.103	0.071	0.083	0.078	0.129	0.115	0.107	0.118	0.103	0.091	0.115	0.092
Std	0.114	0.055	0.090	0.170	0.201	0.168	0.213	0.160	0.196	0.233	0.174	0.151

Panel B: Correlations

Pref	1.00	0.54	0.68	0.40	0.62	0.47	0.34	0.52	0.42	0.34	0.49	0.38
IG Corp	0.54	1.00	0.54	0.29	0.31	0.28	0.21	0.31	0.19	0.16	0.22	0.27
HY Corp	0.68	0.54	1.00	0.63	0.61	0.68	0.60	0.66	0.63	0.59	0.63	0.62
World Ex-US	0.40	0.29	0.63	1.00	0.54	0.82	0.75	0.77	0.75	0.72	0.72	0.82
REIT	0.62	0.31	0.61	0.54	1.00	0.65	0.47	0.75	0.65	0.52	0.77	0.56
MidCap	0.47	0.28	0.68	0.82	0.65	1.00	0.93	0.93	0.93	0.89	0.90	0.93
MidCap Growth	0.34	0.21	0.60	0.75	0.47	0.93	1.00	0.74	0.90	0.95	0.76	0.85
MidCap Value	0.52	0.31	0.66	0.77	0.75	0.93	0.74	1.00	0.82	0.72	0.91	0.88
Rus2000	0.42	0.19	0.63	0.75	0.65	0.93	0.90	0.82	1.00	0.97	0.94	0.81
Rus2000g	0.34	0.16	0.59	0.72	0.52	0.89	0.95	0.72	0.97	1.00	0.83	0.78
Rus2000v	0.49	0.22	0.63	0.72	0.77	0.90	0.76	0.91	0.94	0.83	1.00	0.79
SP500	0.38	0.27	0.62	0.82	0.56	0.93	0.85	0.88	0.81	0.78	0.79	1.00

Table 2: Optimal Portfolios: Unconstrained Case

Optimal portfolios are constructed using different investment opportunity sets. For each opportunity set four cases are considered: (a) Both preferred index and REIT Common index are included; (b) REIT Common index is included, but preferred index is not; (c) Preferred index is included, but REIT Common is not; (d) Neither preferred index not REIT Common are included (no real estate). The table reports expected return, standard deviation, the Sharpe ratio of the market (tangency) portfolio, and the percent improvement in the Sharpe ratio relative to the benchmark case (d) when no REIT common or preferred stock is included. The data is monthly for the period November 1992 through November 2012. The average monthly risk-free rate (1 month T-Bill) is 0.00255. Equity indices are (1) MSCI REIT Preferred Index; (2) MSCI World Ex-US Index; (3) SNL REIT Index; (4) S&P 500 Index; (5) Russell Mid Cap Index; (6) Russell Mid Cap Growth Index; (7) Russell Mid Cap Value Index; (8) Russell 2000; (9) Russell 2000 Growth; (10) Russell 2000 Value. Bond indices are: (1) Barclays Investment Grade Bond Index and (2) Barclay's High Yield Bond Index.

	Market Expected Return	Market St.Dev	Sharpe Ratio	Increase In Sharpe (%)	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)
	Equity Only				Equity and Bonds			
Panel A: All Equity Indices								
Including Prefs & REITS	0.01315	0.0404	0.2622	8.26%	0.0089	0.02203	0.2881	0.38%
Including REITs, No Prefs	0.01715	0.05988	0.2438	0.66%	0.0088	0.02176	0.2872	0.07%
Including Prefs, No REITs	0.01327	0.04093	0.2619	8.13%	0.00894	0.02221	0.2877	0.24%
No REITs, No Prefs	0.01729	0.06085	0.2422	Base	0.00884	0.02189	0.287	Base
Panel B: S&P 500, World Ex-US, MidCap, Russell 2000 (No Midcap V/G, No Russell 2000 V/G)								
Including Prefs & REITS	0.01179	0.04158	0.2223	19.58%	0.00789	0.02096	0.2546	1.56%
Including REITs, No Prefs	0.01651	0.07029	0.1986	6.83%	0.00776	0.0206	0.2528	0.84%
Including Prefs, No REITs	0.01162	0.04087	0.2219	19.37%	0.00783	0.02077	0.254	1.32%
No REITs, No Prefs	0.01714	0.07847	0.1859	Base	0.00758	0.02007	0.2507	Base
Panel C: S&P 500, World Ex-US, MidCap V/G, Russell 2000 V/G (No MidCap, No Russell 2000)								
Including Prefs & REITS	0.01255	0.04139	0.242	10.50%	0.00831	0.0208	0.2766	0.44%
Including REITs, No Prefs	0.0172	0.06656	0.22	0.46%	0.00824	0.02059	0.2759	0.18%
Including Prefs, No REITs	0.01275	0.04235	0.241	10.05%	0.00835	0.02103	0.2757	0.11%
No REITs, No Prefs	0.01729	0.06728	0.219	Base	0.00829	0.02083	0.2754	Base
Panel D (Most Restricted Set): S&P 500, World Ex-US (No MidCap, MidCap V or G, Russell 2000, Russell 2000 V or G)								
Including Prefs & REITS	0.00894	0.03238	0.1973	64.14%	0.00682	0.01794	0.2376	3.85%
Including REITs, No Prefs	0.01026	0.04987	0.1546	28.62%	0.00663	0.01741	0.2343	2.40%
Including Prefs, No REITs	0.00872	0.03138	0.1964	63.39%	0.00671	0.01763	0.2358	3.06%
No REITs, No Prefs	0.00804	0.04568	0.1202	Base	0.00634	0.01654	0.2288	Base

Table 3: Portfolio Allocation Changes (No REIT Common Stock)

Table reports changes in long only optimal portfolio allocations for different levels of risk aversion when REIT common stock is removed from the investor's asset set. Mean changes in allocations are calculated from 15,000 Monte Carlo simulations. Rf is the change in allocation to the risk free asset, IG is the change in allocation to investment grade bonds, HY is the change in allocation to high yield bonds, Equity is the change in allocation to all the other equity asset classes, and Pref is the change in allocation to REIT preferred stock. 95% confidence interval is reported in brackets.

Risk Aversion	Rf	IG	HY	Equity	Pref
0.5	0	0.35	0.44	30.35	10.44
	[0,0]	[0.27,0.43]	[0.35,0.53]	[29.67,31.04]	[9.99,10.88]
1	0	0.39	0.55	25.88	10.12
	[0,0]	[0.31,0.46]	[0.46,0.64]	[25.27,26.49]	[9.73,10.52]
1.5	0	0.38	0.6	21.75	9.68
	[0,0]	[0.31,0.44]	[0.51,0.68]	[21.22,22.28]	[9.34,10.03]
2	0	0.31	0.63	18.12	9.03
	[-0.01,0]	[0.24,0.37]	[0.55,0.71]	[17.66,18.57]	[8.73,9.34]
2.5	0	0.21	0.66	15.08	8.29
	[0,0]	[0.15,0.28]	[0.58,0.74]	[14.69,15.47]	[8.03,8.56]
3	0	0.1	0.67	12.68	7.52
	[-0.01,0]	[0.03,0.16]	[0.6,0.75]	[12.34,13.02]	[7.28,7.76]
3.5	-0.01	-0.04	0.67	10.78	6.81
	[-0.01,0]	[-0.1,0.02]	[0.59,0.74]	[10.49,11.07]	[6.6,7.02]
4	-0.01	-0.2	0.66	9.29	6.19
	[-0.02,-0.01]	[-0.26,-0.14]	[0.59,0.73]	[9.04,9.55]	[6,6.38]
4.5	-0.01	-0.33	0.65	8.11	5.64
	[-0.02,-0.01]	[-0.39,-0.27]	[0.58,0.72]	[7.89,8.33]	[5.47,5.81]
5	-0.02	-0.42	0.64	7.19	5.12
	[-0.03,-0.01]	[-0.48,-0.36]	[0.58,0.7]	[6.98,7.39]	[4.97,5.28]
5.5	-0.03	-0.5	0.63	6.45	4.67
	[-0.04,-0.02]	[-0.56,-0.44]	[0.57,0.69]	[6.26,6.63]	[4.53,4.82]
6	-0.03	-0.54	0.62	5.84	4.27
	[-0.04,-0.02]	[-0.6,-0.49]	[0.56,0.68]	[5.67,6]	[4.14,4.4]

Table 3: Portfolio Allocation Changes (No REIT Common Stock) continued

Table reports changes in long only optimal portfolio allocations for different levels of risk aversion when REIT common stock is removed from the investor's asset set. Mean changes in allocations are calculated from 15,000 Monte Carlo simulations. Rf is the change in allocation to the risk free asset, IG is the change in allocation to investment grade bonds, HY is the change in allocation to high yield bonds, Equity is the change in allocation to all the other equity asset classes, and Pref is the change in allocation to REIT preferred stock. 95% confidence interval is reported in brackets.

Risk Aversion	Rf	IG	HY	Equity	Pref
6.5	-0.03	-0.58	0.61	5.33	3.93
	[-0.05,-0.02]	[-0.63,-0.52]	[0.56,0.67]	[5.18,5.48]	[3.81,4.05]
7	-0.04	-0.6	0.61	4.91	3.63
	[-0.05,-0.02]	[-0.66,-0.55]	[0.55,0.66]	[4.77,5.05]	[3.52,3.75]
7.5	-0.04	-0.61	0.6	4.54	3.37
	[-0.06,-0.03]	[-0.66,-0.55]	[0.55,0.65]	[4.41,4.68]	[3.26,3.48]
8	-0.05	-0.6	0.59	4.23	3.13
	[-0.06,-0.03]	[-0.65,-0.55]	[0.54,0.64]	[4.11,4.36]	[3.03,3.24]
8.5	-0.05	-0.58	0.58	3.96	2.92
	[-0.07,-0.04]	[-0.63,-0.54]	[0.53,0.62]	[3.84,4.08]	[2.82,3.02]
9	-0.06	-0.56	0.56	3.72	2.73
	[-0.08,-0.04]	[-0.61,-0.51]	[0.52,0.61]	[3.61,3.83]	[2.63,2.82]
9.5	-0.07	-0.53	0.55	3.51	2.55
	[-0.08,-0.05]	[-0.57,-0.48]	[0.5,0.59]	[3.41,3.62]	[2.47,2.64]
10	-0.06	-0.51	0.53	3.33	2.4
	[-0.08,-0.05]	[-0.55,-0.46]	[0.49,0.57]	[3.23,3.42]	[2.31,2.48]
10.5	-0.07	-0.48	0.52	3.16	2.26
	[-0.09,-0.05]	[-0.52,-0.44]	[0.48,0.56]	[3.06,3.25]	[2.18,2.33]
11	-0.07	-0.45	0.5	3	2.13
	[-0.09,-0.05]	[-0.49,-0.41]	[0.46,0.54]	[2.91,3.09]	[2.06,2.2]
11.5	-0.07	-0.42	0.48	2.86	2.01
	[-0.09,-0.05]	[-0.46,-0.38]	[0.45,0.52]	[2.78,2.95]	[1.94,2.08]
12	-0.08	-0.39	0.47	2.73	1.91
	[-0.09,-0.06]	[-0.43,-0.36]	[0.43,0.5]	[2.65,2.82]	[1.84,1.97]

Table 4: Portfolio Allocation Changes (No REIT Preferred Stock)

Table reports changes in long only optimal portfolio allocations for different levels of risk aversion when REIT preferred stock is removed from the investor's asset set. Mean changes in allocations are calculated from 15,000 Monte Carlo simulations. Rf is the change in allocation to the risk free asset, IG is the change in allocation to investment grade bonds, HY is the change in allocation to high yield bonds, Equity is the change in allocation to all the other equity asset classes, and REIT Common is the change in allocation to REIT common stock. 95% confidence interval is reported in brackets.

Risk Aversion	Rf	IG	HY	Equity	REIT Common
0.5	0	2.51	1.56	4.65	5.3
	[0,0]	[2.28,2.73]	[1.38,1.74]	[4.36,4.94]	[4.99,5.62]
1	0	3.77	2.33	5.62	5.88
	[0,0.01]	[3.52,4.02]	[2.13,2.53]	[5.35,5.9]	[5.59,6.16]
1.5	0.01	5.25	3.25	6.46	6.27
	[0,0.02]	[4.98,5.52]	[3.03,3.48]	[6.2,6.72]	[6.01,6.53]
2	0.01	6.95	4.23	6.81	6.43
	[0,0.03]	[6.65,7.24]	[3.99,4.47]	[6.57,7.05]	[6.2,6.66]
2.5	0.02	8.78	5.13	6.72	6.35
	[0.01,0.04]	[8.47,9.08]	[4.88,5.38]	[6.51,6.93]	[6.15,6.56]
3	0.04	10.57	5.85	6.33	6.15
	[0.02,0.06]	[10.25,10.89]	[5.6,6.11]	[6.14,6.52]	[5.97,6.33]
3.5	0.05	12.1	6.39	5.76	5.87
	[0.03,0.07]	[11.78,12.42]	[6.13,6.65]	[5.59,5.93]	[5.71,6.03]
4	0.06	13.31	6.72	5.1	5.52
	[0.04,0.09]	[12.98,13.63]	[6.46,6.98]	[4.95,5.25]	[5.38,5.67]
4.5	0.07	14.19	6.83	4.44	5.15
	[0.05,0.1]	[13.87,14.52]	[6.58,7.09]	[4.31,4.58]	[5.02,5.29]
5	0.08	14.77	6.78	3.84	4.79
	[0.05,0.11]	[14.45,15.08]	[6.53,7.03]	[3.72,3.95]	[4.67,4.91]
5.5	0.07	15.05	6.62	3.31	4.45
	[0.04,0.1]	[14.74,15.37]	[6.38,6.86]	[3.21,3.42]	[4.33,4.56]
6	0.05	15.13	6.37	2.86	4.12
	[0.02,0.09]	[14.82,15.43]	[6.14,6.6]	[2.77,2.96]	[4.01,4.23]

Table 4: Portfolio Allocation Changes (No REIT Preferred Stock) continued

Table reports changes in long only optimal portfolio allocations for different levels of risk aversion when REIT preferred stock is removed from the investor's asset set. Mean changes in allocations are calculated from 15,000 Monte Carlo simulations. Rf is the change in allocation to the risk free asset, IG is the change in allocation to investment grade bonds, HY is the change in allocation to high yield bonds, Equity is the change in allocation to all the other equity asset classes, and REIT Common is the change in allocation to REIT common stock. 95% confidence interval is reported in brackets.

Risk Aversion	Rf	IG	HY	Equity	REIT Common
6.5	0.03	15.01	6.07	2.5	3.81
	[0,0.06]	[14.71,15.31]	[5.85,6.29]	[2.42,2.59]	[3.71,3.91]
7	0	14.74	5.75	2.21	3.53
	[-0.03,0.03]	[14.45,15.03]	[5.54,5.96]	[2.13,2.29]	[3.44,3.63]
7.5	-0.03	14.35	5.43	1.98	3.28
	[-0.06,0.01]	[14.07,14.63]	[5.23,5.62]	[1.91,2.05]	[3.19,3.36]
8	-0.06	13.89	5.11	1.79	3.04
	[-0.09,-0.02]	[13.62,14.16]	[4.93,5.3]	[1.73,1.86]	[2.96,3.13]
8.5	-0.09	13.41	4.81	1.65	2.83
	[-0.13,-0.06]	[13.15,13.67]	[4.64,4.99]	[1.58,1.71]	[2.76,2.91]
9	-0.14	12.93	4.53	1.53	2.65
	[-0.17,-0.1]	[12.67,13.18]	[4.37,4.7]	[1.47,1.58]	[2.57,2.72]
9.5	-0.18	12.44	4.28	1.43	2.47
	[-0.22,-0.15]	[12.2,12.69]	[4.12,4.43]	[1.38,1.49]	[2.4,2.54]
10	-0.23	11.97	4.05	1.35	2.32
	[-0.27,-0.19]	[11.74,12.2]	[3.9,4.19]	[1.3,1.4]	[2.25,2.38]
10.5	-0.28	11.52	3.84	1.28	2.18
	[-0.32,-0.24]	[11.29,11.74]	[3.7,3.98]	[1.23,1.33]	[2.11,2.24]
11	-0.32	11.09	3.66	1.22	2.05
	[-0.36,-0.28]	[10.87,11.3]	[3.53,3.79]	[1.18,1.27]	[1.99,2.11]
11.5	-0.36	10.68	3.49	1.17	1.93
	[-0.4,-0.32]	[10.47,10.89]	[3.36,3.61]	[1.12,1.21]	[1.88,1.99]
12	-0.39	10.29	3.33	1.12	1.83
	[-0.43,-0.35]	[10.09,10.49]	[3.22,3.45]	[1.08,1.16]	[1.78,1.88]

Table 5: Changes in Sharpe Ratios

Table reports mean percentage change in Sharpe Ratios caused by the inclusion of given assets. Changes in Sharpe Ratios are calculated as the average percentage change in Sharpe Ratio calculated over 15,000 Monte Carlo simulations. Preferred refers to the case where REIT preferred stock is added to the asset set, REIT Common refers to the case where REIT common stock is added to the asset set, and Preferred & REIT Common refers to the case where both REIT preferred and common stock are added to the asset set. In each case the asset set includes the risk free asset, investment grade bonds, high yield debt, and the other equity asset classes. 95% confidence intervals are reported in brackets.

Risk Aversion	Preferred	REIT Common	Preferred & REIT Common
0.5	7.87	-0.58	10.2
	[7.46,8.28]	[-0.8,-0.36]	[9.6,10.79]
1	7.57	-0.59	9.73
	[7.23,7.91]	[-0.77,-0.42]	[9.18,10.28]
1.5	7.29	-0.49	9.27
	[7.00,7.58]	[-0.63,-0.35]	[8.75,9.79]
2	6.88	-0.26	8.74
	[6.62,7.13]	[-0.38,-0.15]	[8.24,9.24]
2.5	6.35	-0.02	8.13
	[6.12,6.57]	[-0.11,0.08]	[7.65,8.61]
3	5.77	0.22	7.51
	[5.56,5.97]	[0.14,0.3]	[7.04,7.98]
3.5	5.22	0.43	6.95
	[5.04,5.4]	[0.36,0.51]	[6.49,7.41]
4	4.75	0.61	6.48
	[4.58,4.91]	[0.54,0.68]	[6.02,6.93]
4.5	4.35	0.75	6.1
	[4.2,4.51]	[0.68,0.81]	[5.66,6.55]
5	4.04	0.86	5.83
	[3.9,4.19]	[0.8,0.92]	[5.39,6.28]
5.5	3.81	0.95	5.64
	[3.67,3.94]	[0.89,1.01]	[5.2,6.08]
6	3.63	1.01	5.51
	[3.5,3.76]	[0.95,1.08]	[5.08,5.95]

Table 5: Changes in Sharpe Ratios continued

Table reports mean percentage change in Sharpe Ratios caused by the inclusion of given assets. Changes in Sharpe Ratios are calculated as the average percentage change in Sharpe Ratio calculated over 15,000 Monte Carlo simulations. Preferred refers to the case where REIT preferred stock is added to the asset set, REIT Common refers to the case where REIT common stock is added to the asset set, and Preferred & REIT Common refers to the case where both REIT preferred and common stock are added to the asset set. In each case the asset set includes the risk free asset, investment grade bonds, high yield debt, and the other equity asset classes. 95% confidence intervals are reported in brackets.

Risk Aversion	Preferred	REIT Common	Preferred & REIT Common
6.5	3.51	1.07	5.44
	[3.39,3.64]	[1.01,1.13]	[5,5.88]
7	3.43	1.11	5.41
	[3.31,3.55]	[1.05,1.17]	[4.97,5.84]
7.5	3.38	1.14	5.4
	[3.26,3.5]	[1.08,1.2]	[4.97,5.84]
8	3.36	1.16	5.42
	[3.24,3.47]	[1.1,1.22]	[4.98,5.85]
8.5	3.35	1.18	5.44
	[3.23,3.46]	[1.12,1.24]	[5.01,5.88]
9	3.34	1.2	5.48
	[3.23,3.46]	[1.14,1.26]	[5.04,5.91]
9.5	3.35	1.21	5.51
	[3.23,3.47]	[1.15,1.27]	[5.08,5.94]
10	3.36	1.22	5.55
	[3.25,3.48]	[1.16,1.28]	[5.11,5.98]
10.5	3.37	1.23	5.58
	[3.26,3.49]	[1.17,1.29]	[5.15,6.02]
11	3.39	1.24	5.62
	[3.27,3.5]	[1.18,1.3]	[5.18,6.05]
11.5	3.4	1.24	5.65
	[3.29,3.52]	[1.18,1.3]	[5.22,6.08]
12	3.42	1.25	5.68
	[3.3,3.53]	[1.19,1.31]	[5.24,6.11]